Public Equity Markets and Aggregate Productivity*

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Abstract

We study how differences between publicly-listed and privately-owned firms contribute to aggregate productivity. Using European firm-level data, we document that publicly-listed firms are larger and more productive than private firms. We show that firms substantially increase assets and employment around the time of their initial public offering (IPO). In order to quantify the aggregate importance of these differences and growth, we build a firm dynamics model, where both public and private firms make investments to improve productivity. Private firms endogenously choose to conduct IPOs, which transforms the firm into a public firm. Public firms face lower capital costs and capital costs scale less with productivity, incentivizing investment. We find that the existence of public equity markets boosts GDP by 0.5% in Italy and by 6.0% in France. Differences in the institutions of public equity markets explain between one-half and three-quarters of the France-Italy productivity gap. We also use the quantitative model to measure the productivity losses due to differences in capital costs between public and private firms, and study a policy experiment where the government subsidizes IPO costs to boost aggregate productivity.

Keywords: Aggregate Productivity, Financial Frictions, Firm Dynamics, Public Equity Markets, Economic Growth

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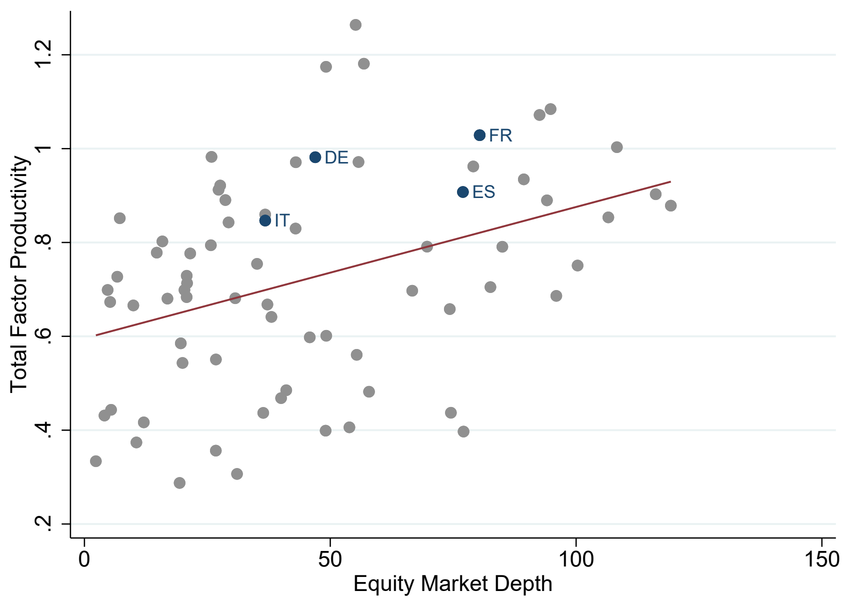
1 Introduction

While only 0.06% of U.S. firms are listed on public equity markets, they account for 31% of private sector employment and 41% of sales (Asker et al., 2015). Across countries, Figure 1 shows that deeper public equity markets, measured as the total market capitalization of firms divided by GDP, are correlated with higher aggregate total factor productivity. Despite their importance, public equity markets are an under-emphasized dimension of financial development, relative to work on credit constraints faced by young firms (e.g., Buera et al., 2015). In this paper, we fill this gap by studying the importance of public equity markets for cross-country income and productivity differences.

Figure 1: Equity Market Depth and Aggregate Productivity



Notes: Equity market depth is calculated as the market capitalization of publicly listed firms divided by GDP, which is taken from the World Bank. Total factor productivity is calculated as a Solow residual from the Penn World Tables data (version 10.0). Each point represents a single country's average over the years 2000-2019. We highlight in blue the four countries studied in this paper: France (FR), Italy (IT), Germany (DE), and Spain (ES).

We start by documenting facts about the differences between publicly-listed firms and privately-held firms. Using the Orbis dataset, we show that in four major European economies: France, Germany, Italy and Spain, publicly-listed firms are larger, more productive, and better capitalized than private firms. Of course, these differences could be driven by selection, if larger, more productive, or better capitalized firms are more likely to select into being publicly-listed.

To investigate this, we follow private firms that transition from being private to listing publicly, through initial public offerings (IPOs). By running a two-way fixed effect regression, we document the average growth in assets and employment over an IPO. On average, French firms that conduct an initial public offering increase their asset levels by 95 log points (approximately a 150% increase) and their employment by 44 log points (approximately a 50% increase) in the year after they conduct an IPO. These effects are relative to other private firms who will one day go public, which minimizes the role for selection to drive these results. The estimated effects in Germany, Italy, and Spain are even larger. These results suggest that the transition from being private to being public is a transformative phase of growth for firms, and are consistent with substantially different degrees of financial frictions that impact public and private firms.

In order to understand the aggregate implications of these differences, we develop a firm dynamics models that incorporates public and private firms. Both public and private firms make risky investments to improve productivity. Productivity follows a technology ladder in which firms invest to stochastically move to higher rungs, with higher productivities. Firms also face a risk of moving to a lower rung, through an obsolescence shock, and exit if they move below the entry rung, increasing the risk of exit for low-productivity firms. In addition, firm productivity depends on a permanent firm-specific component, an idiosyncratic component that varies over time, and a corporate structure component that depends on whether the firm is public or private.

The main novelty of our model is to incorporate an endogenous initial public offering (IPO) decision. Private firms become public firms by paying a fixed cost to conduct an initial public offering (IPO), which transforms the firm from a private to a public firm. Firms are sold in IPOs for their fair-market value as a public firm, minus the fixed IPO cost. The primary difference between public and private firms is in the degree of financial frictions they face in acquiring capital. We model these financial frictions in a reduced form as capital wedges. Capital wedges increase as firms become more productive, consistent with empirical evidence that we document on the relationship between firm-level capital stocks and productivity. Relative to private firms, public firms face lower capital wedges that are less correlated with productivity. Additionally, we allow for public firms to differ from private firms in a residual productivity component, which can either increase (e.g., due to improved management incentives) or decrease (e.g., agency costs of management) productivity.

The model equilibrium features a non-degenerate distribution of both public and private firms. As in the data, public firms tend to be larger and more productive than private firms since the fixed IPO cost prevents low-productivity firms from conducting IPOs. Aggregate productivity depends on capital misallocation (both between and within public and private firms) and the productivity of firms, determined by firm investment and IPO decisions. Public equity markets directly affect aggregate productivity through both capital misallocation, through differential capital wedges, and firm productivity, through the residual component of productivity. Public equity markets also indirectly affect aggregate productivity through incentivizing investment by firms (i.e., because of the option value of IPOs).

We calibrate four economies to match data from France, Germany, Italy, and Spain. These four countries are our focus as they are major European economies where our data coverage is the best. We use information from event studies of private firm transforming into public firms through IPOs to discipline the relative degree of financial frictions between private and public firms. We also include detailed information about the cross-section of employment and employment growth to discipline firm life-cycles in the model, since growth over the firm's life cycle impacts the relative size of public and private firms. We target the firm and employment share of small firms (fewer than ten employees) to discipline the share of firms that could potentially experience IPOs since this has important implications in our quantitative experiments. Finally, we include several moments related to the aggregate importance of public firms, such as the employment share of public firms, to discipline parameters related to characteristics of public and private firms. The quantitative model is able to replicate non-targeted features of the data, such as the firm and employment distribution across employment bins.

We use the quantitative model to examine the aggregate importance of public equity markets. In the first experiment, we measure the contribution of public equity markets to output. In each of the four economies we compare aggregate output in the benchmark economies with counterfactual economies in which the public equity market is shut down. We find that the equity market value varies from around 0.5% of GDP in Italy (the shallowest equity market) to around 6% of GDP in France (the deepest equity market). The value is smaller than the overall contribution of public firms to output, which is around one-quarter (or four times the measured equity market value) in the benchmark French economy. This is due to the fact that the public firms are not removed from the economy, but instead become large, productive private firms in the counterfactual economy. The value of public equity markets is determined more through their impact on capital misallocation and the indirect effect of incentivizing firm investment.

In the second experiment, we examine the role of public equity markets for explaining cross-country productivity differences. We find that moving IPO costs from the French benchmark economy to match levels estimated for Italy and Spain explains around one-half to three-quarters of the output per capita and TFP gaps observed in the data. These differences are driven by shallower public equity markets discouraging firm investment and hurting the allocative efficiency of capital. In the third experiment, we assess the importance of within-country differences in financial frictions between public and private firms. We adjust capital costs such that private firms face the same lower capital cost elasticity as public firms. This leads to a large increase in output per capita and TFP of between 19% and 35%. The increase is driven by increased allocative efficiency of capital and higher investment by incumbent and entrant firms.

Finally, we use the model to examine how policy could boost aggregate productivity by expanding the depth of the public equity market using the economy calibrated to French data. We consider a budget-neutral policy in which subsidies to IPO costs are financed through distortionary taxes on firm profits. Subsidies to IPO costs allow more firms to go public and encourages investment. On the other hand, taxing firm profits discourages investment by all public and private firms. The experiment highlights an important dynamic in the model between public and private firms. While public firms account for a large share of employment and output, they account for a relatively small share of firms, limiting the fiscal cost of supporting these firms. The fiscal cost is further limited by the one-time nature of IPO costs. Consequently, the results show that supporting public equity markets can have relatively small costs but large aggregate productivity benefits. We also find that the benefits are potentially larger in countries with shallower public equity markets from comparing the policy in Spain and France.

Related literature. Our paper contributes to several literatures. Firstly, our paper relates to the literature examining the relationship between financial markets, firm heterogeneity and development. A large literature has linked financial underdevelopment in low-income countries with capital misallocation (e.g., Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014) and the lack of technology adoption (e.g., Buera and Shin, 2013; Moll, 2014; Cole et al., 2016). Midrigan and Xu (2014) show that capital misallocation may be additionally harmful to the extent that it prevents firms from adopting advanced technologies. Cole et al. (2016) examine a richer set of technologies and financial contracts showing how technologies with convex payoff profiles can be restricted by lack of financial development leading to flatter firm life-cycle profiles (as documented by Hsieh and Klenow, 2014). We add to this literature by studying public equity markets as another dimension of financial development. Public equity markets determine capital costs of the most productive firms in the economy and incentivize investment by allowing high productivity firms to rapidly expand their scale. In this regard, public equity markets provide another factor that may explain the dynamism of firms in more financially developed countries.

The differences in capital wedges faced by private and public firms also relates us to the closely-related literature on misallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). We document empirically that public firms face both lower overall capital wedges and a lower slope of capital wedges as they become more productive. This highlights a channel in which the most economically important firms can lower capital wedges and move towards their first-best allocation. Additionally, we find important dynamic effects in which public equity markets incentivize firm investment, relating our paper to a large literature examining the dynamic effects of misallocation (e.g., Gabler and Poschke, 2013; Bento and Restuccia, 2017; Akcigit et al., 2021; Ayerst, 2021).

Secondly, our paper relates to the literature on the macroeconomic importance of entrepreneurship. The literature generally focuses on the importance of credits constraints for new entrepreneurs (e.g., Buera et al., 2011) and risk taking (Vereshchagina and Hopenhayn, 2009; Robinson, 2021). In contrast, we examine how public equity markets affect risky investment and firm dynamics of more mature firms, and develop a model that allows us to examine differences in public and private firms. Given this, we model credit constraints as a captial wedge that depends on firm-level productivity causing larger, more-productive firms to be more severely constrained. In a related paper, Peter (2021) examines the impact of partial firm ownership, through sales of equity, on wealth inequality. By contrast, we take the decision to sell through an IPO as a binary outcome, which allows us to highlight differences in public and private firms, such as difference in capital costs. We also model richer firm life-cycle dynamics that we find are key to disentangling the implications of public equity markets for aggregate productivity.

Several papers study different elements of private firm organization. In particular, Glover and Short (2018) study how limited liability for incorporated entrepreneurs encourages entrepreneurial risk taking, relative to unincorporated entrepreneurs. Dyrda and Pugsley (2019) study the trade-offs between S-corporations and C-corporations, and document how the rise in S-corporations has contributed to the rise in income inequality in the US. By contrast, we focus on the differences between privately-held firms and firms that are listed on public stock exchanges.

Finally, our paper relates to the corporate finance literature comparing the behaviour of public and private firms. Several papers examine how public and private firms differ in their propensity to invest (Asker et al., 2015) or innovate (e.g., Bernstein, 2015; Acharya and Xu, 2017). Asker et al. (2015) find that public firms invest less than a selected group of private firms and attribute this to excessive "short-termism" due to the need for managers of public firms to meet quarterly earnings expectations. Bernstein (2015) compares the innovation activities of firms that complete or do not complete an initial public offering (IPO). He instruments for IPO completion using the NASDAQ fluctuations in between the initial IPO filing and the completion of an IPO, and finds that the quality of internal innovation declines following an IPO but that public firms are able to acquire external innovation more easily. The differing behaviours of public and private firms have important implications given the recent decline in the number of listed public firms, as documented by Gao et al. (2013) and Doidge et al. (2017).

2 Empirical Evidence on Public and Private Firms

In this section, we provide empirical evidence on the differences between public and private firms. We start by discussing the construction of our dataset. Using this dataset, we then show that, within countries, public firms are substantially larger, more productive, and better capitalized than private firms. We also document that firms experience large increases in assets and employment as they transition from being private to public through an initial public offering (IPO). We view these facts as informative of the importance of public equity markets and so use them to discipline our model in Section 4.

2.1 Data

Orbis. In order to study differences between public and private firms, we need data on both firm types over time. To this end, we use the Orbis dataset, which is an annual panel data set produced by Bureau van Dijk containing rich information about company financials, ownership, legal structure, and employment. Bureau van Dijk collects this information from government business registries, firm financial statements, and other public sources. Orbis has two key advantages for this paper: first, the data contains comprehensive information about both private and public firms in a comparable format, and second, this information is comparable across countries with differening levels of equity market depth.

We focus on France, Germany, Italy, and Spain from 2000 to 2019. We exclude earlier periods as the data coverage is worse. We exclude later periods to avoid including the COVID pandemic and Russia's invasion of Ukraine. The four countries represent the largest economies in the Euro Area and capture a range of equity market depths. Focusing on these Euro Area economies allow us to use data from Eurostat to weight firms, improving the national representativeness of the data.

Eurostat. We supplement the Orbis dataset with data from the Business Demography (BD) and Structural Business Statistics (SBS) databases from Eurostat. We follow Kalemli-Ozcan et al. (2022) in using the BD and SBS databases to construct nationally representative firm weights. The Orbis data tends to under-sample small firms making the weights necessary in order to appropriately characterize the entire distribution of firms. We extend the construction of firm weights in Kalemli-Ozcan et al. (2022) to allow for more granularity in the firm bins. Appendix A.1 describe the construction of firm weights in more detail. In addition, we construct moments related to firm entry, exit, and the economic activity of firms in different size bins for the calibration using Eurostat SBS data. These measures of entry and exit are useful because we do not directly observe firm entry or exit in the Orbis dataset. The first or last year a firm is in the dataset may simply correspond to the first or last year its data is collected in Orbis, rather than true entry or exit.

Aggregate data sources. We use the Penn World Tables (v10.0) to construct aggregate measures of real productivity across countries. We also use the World Bank's data on equity market depth, measured as total market capitalization divided by GDP, across countries.

2.2 Differences Between Public and Private Firms

In the cross section, public firms are larger than private firms across all four countries. Table 1 reports summary statistics on employment for our four countries. Appendix Tables 13 and 14 show that public firms are also larger than private firms in terms of both output and assets.

Table 1: Employment

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	1	6	9	6	1,376	631,007
Germany	1	6	12	10	285	114,594
Italy	1	6	9	6	803	659,277
Spain	1	6	9	6	1,906	568,653

Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data.

Public firms have disproportionately more employees compared to private firms, with the average public firm being several orders of magnitude larger than the average private firm. As is well-known, the firm-size distribution is extremely skewed with the vast majority of firms employing a small number of workers. These small firms are disproportionately private, while public firms are, on average, substantially larger.

Public firms are also disproportionately productive. Table 2 shows summary statistics on total factor productivity across our four countries.[1] On average, public firms are substantially

[1] Total factor productivity is computed as $TFP_{j,t} = y_{j,t}/(k_{j,t}^{\alpha}\ell_{j,t}^{1-\alpha})^{1-\gamma}$ using $\alpha = 0.33$ and $\gamma = 0.85$,

Table 2: TFP

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	5.7	11.9	26.6	14.7	44.0	458,906
Germany	8.1	20.7	64.4	30.7	31.6	25,773
Italy	3.7	11.5	32.2	16.0	36.1	435,624
Spain	2.4	6.3	14.6	7.8	46.6	456,364

Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data.

more productive than their private counterparts[2]. In Appendix Table 15, we show that this pattern also holds for sales-per-employee, a simple measure of productivity, and in appendix figure 8 we show the full distribution of TFP for public and private firms.

Table 3: Capital-Output Ratios

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	0.02	0.20	1.11	0.44	3.46	492,126
Germany	0.01	0.16	1.73	0.69	9.04	26,161
Italy	0.03	0.24	1.95	0.88	20.56	431,745
Spain	0.03	0.38	2.62	1.25	3.38	466,168

Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data.

In Table 3, we show that public firms also have much higher capital-output ratios across countries. This is consistent with the idea that public firms face fewer financial frictions or a lower cost of capital than their private counterparts. In appendix figure 9 we compare the full distributions of Capital Output ratios for public and private firms.

2.3 Firm Changes Around IPOs

In this subsection, we study how firms change as they transition from private to public, through an initial public offering (IPO).

We use a simple event study framework that characterizes how assets and employment change around the time of the firm's IPO. Specifically, we estimate the following equation:

consistent with the model in the following section. See appendix A.1 for details.

[2] This pattern is weaker in Germany. However, the German data is disproportionately missing sales and material cost data for private firms with low levels of employment, which tend to be less productive.

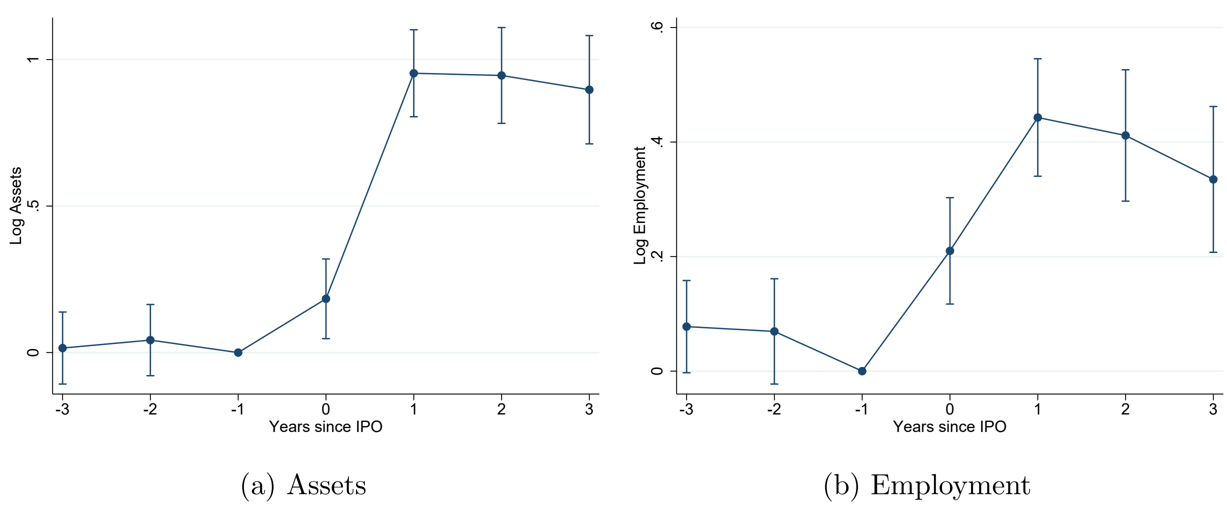
$$\log(y_{j,t}) = \sum_{\substack{k=-3 \\ k\neq -1}}^{3} \beta_k D^k_{j,t} + \alpha_j + \alpha_t + \epsilon_{j,t}, \tag{1}$$

where we regress the log of outcome y for firm j in year t, on a set of dummy variables $D^k_{j,t}$. These variables equal one if firm j conducted an IPO in year $t-k$, and are zero otherwise. The coefficients of interest are a series of coefficients $\{\beta_k\}$ that trace out average log changes in outcome y in the years just before and just after an IPO. α_j is a vector of firm fixed effects, α_t is a vector of year fixed effects, and $\epsilon_{j,t}$ is an error term.

We run regression (1) on the set of private firms that ever go through an IPO, intentionally excluding private firms that we never observe going public as well as firm-year observations of public firms that have been public for more than three years. This sample gives us a difference-in-difference strategy which compares the group of treated firms, who undergo an IPO in the year $t-k$, to a group of control firms, who are private firms that will become public in more than three years. The coefficient β_k then describes the average change in the log outcome experienced by firms going public in year $t+k$. We balance the panel for the three years before and after an IPO to ensure that effects are not driven by differences in data availability.

Figure 2 reports the average log changes in fixed assets and employment around firm IPOs in France. For brevity, we focus on France and summarize the results for other countries in Table 4. Details for the remaining countries are reported in Appendix A.4.

Figure 2: Assets and Employment over IPOs in France



(a) Assets (b) Employment

We find that these IPO events have transformative effects on these firms. Assets increase by 0.95 log points relative to their pre-IPO asset level. Employment growth is less dramatic, but substantial. One year after an IPO, firms have increased their employment by 0.44 log points relative to the year before an IPO.

It is important to note that these estimates are not causal. Private firms in this environment are choosing to become public in year $t-k$ and likely anticipate doing so in the years before. That being said, the absence of significant pre-trends and the fact that the comparison group is private firms who we observe going public at a later date gives us confidence that these results are not mostly driven by selection.

Table 4: Growth over IPOs across Countries

Country	Assets	Employment
France	0.95	0.44
Germany	1.00	0.62
Italy	1.28	0.86
Spain	2.13	0.86

Notes: This table reports the estimates of β_1 for each country. These coefficients correspond to the average log change in assets and employment in the year after the IPO.

Table 4 shows that firms increase their assets by between 0.95 and 2.13 log points from the year prior to to the year following their IPO. These numbers represent very large increases in the asset levels of these firms. In one respect these large increases are unsurprising, as the stated goal of most IPOs is to raise additional capital. On the other hand, the power of public equity markets are the way they make a vast quantity of capital readily available to firms in a short period of time.

Employment also increases by between 0.44 and 0.86 log points to the year after the IPO. This high rate of employment growth indicates that IPOs are not just a financial transaction, but that these IPOs materially affect real economic outcomes of the firm.

These large changes in firm assets and employment in such a short window of time, suggest that the transition from private to public status is an important period of firm's lifecycle. In order to quantify how important these transitions can be for aggregate productivity, in the next section we build a model of firm dynamics.

3 Model

In order to understand the aggregate implications of public equity markets, we develop a model of firm dynamic that incorporates public equity markets. Both public and private firms grow over time through productivity-increasing investments. Private firms become public firms through initial public offerings (IPOs), which allows the firm to immediately receive the fair-market value of the public firm by paying a fixed IPO cost. The model endogenizes the dynamic tradeoffs and forward-looking decisions made by firms, which are important for understanding the aggregate implications of public equity markets. In the following sections, we quantify the model to examine the cross-country productivity implications of public equity markets.

3.1 Economic Environment

The economy is populated by an endogenous mass M of firms and a representative household. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

Production. The final good is produced by an endogenous mass M of heterogeneous firms, indexed by j, that differ in productivity $z^{\kappa}_{j,t}$ and corporate structure $\kappa \in \{Pri, Pub\}$. Firms' corporate structure can either be privately owned ($\kappa = Pri$) or publicly listed ($\kappa = Pub$). Both types of firms produce by renting capital $k_{j,t}$ and hiring labor $\ell_{j,t}$ according to:

$$y^{\kappa}_{j,t} = (z^{\kappa}_{j,t})^{1-\gamma} \left(k^{\alpha}_{j,t}\ell^{1-\alpha}_{j,t}\right)^{\gamma}. \tag{2}$$

The productivity of a firm j is given by:

$$\ln z^{\kappa}_{j,t} = \ln a_j + n_{j,t} \ln \lambda + \ln \eta^{\kappa} + v_{j,t}. \tag{3}$$

The first component a_j is a permanent component of productivity corresponding to the firm's fundamental type. Firms are either low types, with $a_j = a_l$, or high types, with $a_j = a_h$ (with some abuse of notation as subscripts l and h refer to firm types and not firm indexes j). Types are determined at entry with probability ω_{a_l} and ω_{a_h}. The second component $n_{j,t} \ln \lambda$ is a firm-specific production technology that can be improved through investment, which is discussed below. The third component η^{κ} captures the residual productivity differences between private $\eta^{Pri} = 1$ and public firms $\eta^{Pub} = \eta$. We allow for the firm's corporate structure to impact productivity through this parameter. The value of η could, for example, capture agency costs associated with aligning management and shareholder incentives if $\eta < 1$ (as in Peter, 2021) or improved accountability of management and workers if $\eta > 1$. The final component of productivity $v_{j,t}$ captures an idiosyncratic component that is drawn from distribution $\Phi(v) = N(0, \sigma^2_v)$, before production and hiring decisions occur.

Firm investment. The technology component of productivity $\lambda^{n_{j,t}}$ takes values along a grid $n_{j,t} \in \{0, 1, 2, ..., N\}$ where firms invest to sequentially move to more productive technologies (i.e., from n to $n+1$), building on models of innovation along quality ladders (e.g., Aghion and Howitt, 1992; Klette and Kortum, 2004; Akcigit and Kerr, 2018). We assume that only high type firms can improve technology, based on the preponderance of small low-growth firms in the firm-size distribution. Firms choose a rate x to improve their production technology by investing:

$$\psi x^{\zeta} \lambda^{n\phi},$$

where $\psi > 0$ is the level of investment cost, $\zeta > 1$ is the curvature of investment cost, and $\lambda^{n\phi}$ is a scaling term that captures that it is more costly to improve better technologies. The parameter ϕ dictates how quickly investment costs scale with firm productivity.[3]

Firms receive obsolescence shocks at rate χ that moves firms from technology n to $n-1$, or cause firms to exit if they are at $n = 1$. For simplicity, we assume that only firms that do not receive obsolescence shocks can improve technology in the current period t and that improvements occurs at rate $x/(1-\chi)$, such that x remains the overall probability that firms improve technology.

Entry and exit. New firms enter through costly investment. Firms enter at rate u by paying a cost:

$$\psi_E u^{\zeta}$$

where $\zeta \geq 1$ captures the curvature of investment in entry. New firms enter as private firms with technology $n = 1$. New firms are high types with probability ω_{a_h} and low types with probability ω_{a_l}.

Firms exit at exogenous rate $\nu \in (0, 1)$ or when they receive an obsolescence shock at production technology $n = 1$. The obsolescence shock creates an additional risk for small firms, consistent with the data and as documented in the literature (e.g., Klette and Kortum, 2004).

[3]In the quantitative analysis, we set the parameter ϕ such that it offsets the impact of capital wedges on the change in firm productivity. This brings us closer to the Gibrat's law benchmark (where firm growth is independent of firm productivity) commonly assumed in the literature. In a model without capital wedges (discussed later), public equity markets, and obsolescence shocks, setting $\phi = 1$ would result in investment costs scaling proportionally with profits leading to Gibrat's law holding in equilibrium.

Households and preferences. There is a representative household with unit mass. Preferences are given by:

$$U([C_t]_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta}, \tag{4}$$

where $\beta \in (0,1)$ is the utility discount rate, and $\theta > 0$ describes the risk aversion of households. Households accumulate capital through investment according to $K_{t+1} = (1-\delta)K_t + I_t$, where $\delta \in (0,1)$ is the depreciation rate of capital and I_t is capital investment. Household members supply labor inelastically.

3.2 Market Structure and Timing

Households own the mutual fund, which is the final owner of public firms, private firms, and the stock of capital. Firms are perfectly competitive and produce a homogeneous final good, which we take as the numeraire.

Firm ownership. A firm j with corporate structure $\kappa \in \{Pri, Pub\}$ has value V_j^{κ}. All firms enter as private firms and become public firms through paying a fixed IPO cost f, which could capture, for example, the costs of going to the market, informing investors, and hiring underwriters. A private firm j that is sold through an IPO is immediately bought by a mutual fund at its market value V_j^{Pub}, implying that the net sales value is $V_j^S = \mathbb{E}[V_j^{Pub}] - f$.

In addition to the IPO cost, private firms have time-varying idiosyncratic preferences over remaining private or selling through an IPO described by i.i.d. random variables ε^{Pri} and ε^S. The idiosyncratic preferences could be thought of as capturing any additional labor or stress associated with operating the firm or the preference of the firm's manager for remaining their own boss (as documented by Hurst and Pugsley, 2016). Both preferences are drawn from a Frechet distribution with shape parameter μ.

Capital wedges. The key difference between public and private firms is access to capital markets. We model capital markets indirectly through idiosyncratic capital wedges $\tau_{j,t}^{\kappa}$. We assume that firm-level capital wedges depend both on the firm's corporate structure κ and productivity $z_{j,t}$, which allows us to capture that larger firms have different access to capital. Specifically, we assume that capital wedges are described by:

$$\ln \tau_{j,t}^{\kappa} = \ln \bar{\tau}^{\kappa} + \rho^{\kappa} \ln z_{j,t}. \tag{5}$$

We borrow from the misallocation literature (e.g., Restuccia and Rogerson, 2008; Hsieh and

Klenow, 2009) by assuming that the indirect wedge acts as stand-ins for multiples factors that may affect firm capital inputs. The above representation is useful in that it allows for more productive firms to face higher capital costs, if ρ^{κ} is larger, which aligns with our goal of examining the transition of medium and large private firms into public firms. For market clearing, we assume that the revenues from the tax are collected by a passive government and then redistributed to households as a non-distortionary lump-sum transfer.

The assumption of credit constraints on large firms also follows others (e.g., Gopinath et al., 2017; David and Venkateswaran, 2019) who find credit restrictions on large firms empirically important. In contrast, the standard borrowing constraints based on wealth tend to only be restrictive to young firms as firms are incentivized to save out of the constraints relatively quickly (e.g., Moll, 2014; Cole et al., 2016).

Timing. Within each period, the timing is as follows: (i) Realization of exogenous exit shocks; (ii) Realization of technology improvements and obsolescence shocks, including exit; (iii) New firms enter; (iv) Realization of preference shocks ε, IPO decision and firm sales; (v) Realization of productivity shock v; (vi) Production; (vii) Investment and consumption decisions are made.

3.3 Equilibrium

We focus on the stationary Markov Perfect Equilibrium. We drop t subscripts where it does not create confusion.

3.3.1 Production and Resource Allocation

The household's Euler Equation implies that $1 + R = 1/\beta$, which implies that the cost of capital is equal to $r = R + \delta$.

Rather than index firms by j, we refer to firms by their type consisting of their corporate structure κ, permanent productivity a, production technology n, and idiosyncratic shock v. The static problem of firms is to choose labour and capital to maximize operating profits in the current period. The static problem is:

$$\pi^{\kappa}_{a,n}(v) = \max_{k,\ell} z^{\kappa}_{a,n}(v)^{1-\gamma} \left(k^{\alpha}\ell^{1-\alpha}\right)^{\gamma} - \tau^{\kappa}_{a,n}(v) r k - w\ell.$$

Demand for capital and labour are given by:

$$\ell^{\kappa}_{a,n}(v) = \left[\left(\frac{\gamma(1-\alpha)}{w}\right)\left(\frac{\alpha}{(1-\alpha)}\frac{w}{\tau^{\kappa}_{a,n}(v)r}\right)^{\alpha\gamma}\right]^{1/(1-\gamma)} z^{\kappa}_{a,n}(v), \tag{6}$$

$$k^{\kappa}_{a,n}(v) = \left[\left(\frac{\gamma\alpha}{\tau^{\kappa}_{a,n}(v)r}\right)\left(\frac{(1-\alpha)}{\alpha}\frac{\tau^{\kappa}_{a,n}(v)r}{w}\right)^{\gamma(1-\alpha)}\right]^{1/(1-\gamma)} z^{\kappa}_{a,n}(v). \tag{7}$$

Firm output is equal to

$$y^{\kappa}_{a,n}(v) = \gamma^{\frac{\gamma}{1-\gamma}}\left[\left(\frac{\alpha}{\tau^{\kappa}_{a,n}(v)r}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}}\right] z^{\kappa}_{a,n}(v).$$

3.3.2 Firm Value, Investment, and IPO Decisions

Static firm profits are equal to $\pi^{\kappa}_{a,n}(v) = (1-\gamma)y^{\kappa}_{a,n}(v)$. The dynamic problem of the firm is to choose investment to maximize firm value and to choose whether to initiate an initial public offering (IPO). The dynamic problem of a public firm is given by:

$$\begin{aligned} V^{Pub}_{a,n}(v) = \max_{x\geq 0}\ & \pi^{Pub}_{a,n}(v) - \psi x^{\zeta}\lambda^{n\phi} \\ & + \frac{1-\nu}{1+R}\mathbb{E}_{v'}\left[V^{Pub}_{a,n}(v') + x\left[V^{Pub}_{a,n+1}(v') - V^{Pub}_{a,n}(v')\right] + \chi\left[V^{Pub}_{a,n-1}(v') - V^{Pub}_{a,n}(v')\right]\right]. \end{aligned} \tag{8}$$

The solution to the public firm's problem is:

$$x^{Pub}_{a,n} = \left[\frac{1-\nu}{1+R}\frac{\mathbb{E}_{v'}\left[V^{Pub}_{a,n+1}(v') - V^{Pub}_{a,n}(v')\right]}{\psi\zeta\lambda^{n\phi}}\right]^{\frac{1}{\zeta-1}},$$

where investment does not depend on the firm's current idiosyncratic productivity shock v. Investment is increasing in the relative increase in firm value from moving from production technology n to $n+1$. For the case where obsolescence shocks are zero (i.e., $\chi = 0$), the relative value of improving technology would be proportional to the relative increase in profitability of the new technology.

The dynamic problem of a private firm is given by:

$$\begin{aligned} V^{Pri}_{a,n}(v) = \max_{x\geq 0}\ & \pi^{Pri}_{a,n}(v) - \psi x^{\zeta}\lambda^{n\phi} \\ & + \frac{1-\nu}{1+R}\left[\tilde{V}^{Pri}_{a,n} + x\left[\tilde{V}^{Pri}_{a,n+1} - \tilde{V}^{Pri}_{a,n}\right] + \chi\left[\tilde{V}^{Pri}_{a,n-1} - \tilde{V}^{Pri}_{a,n}\right]\right]. \end{aligned} \tag{9}$$

The main difference between public and private firm value is that private firms consider the future value $\tilde{V}^{Pri}_{a,n}$, which contains the option value of going through an IPO, when making decisions. This option value $\tilde{V}^{Pri}$ is equal to:

$$\tilde{V}^{Pri}_{a,n} = \mathbb{E}_\varepsilon \left[\max_{d \in \{0,1\}} (1-d)\varepsilon^{Pri}[\mathbb{E}_{v'} V^{Pri}_{a,n}(v')] + d\varepsilon^S V^S_{a,n} \right], \tag{10}$$

where $\varepsilon = (\varepsilon^S, \varepsilon^{Pri})$, $d = 1$ reflects the firm's decision to go public, $V^S_{a,n}$ is the value of selling a firm with production technology n, where $V^S_{a,n} = \mathbb{E}_{v'}[V^{Pub}_{a,n}(v')] - f$. The investment decision of private firms is similar to public firms with $\tilde{V}^{Pri}_{a,n}$ replacing the private value, highlighting how future IPOs can affect firm investment decisions.

The probability that a type a private firm with technology n goes public is:

$$s_{a,n} = \Pr[d_{a,n}(\varepsilon) = 1] = \begin{cases} 0 & \text{if } V^S_{a,n} < 0 \\ \frac{(V^S_{a,n}(v))^\mu}{(V^S_{a,n}(v))^\mu + (\mathbb{E}_v V^{Pri}_{a,n}(v))^\mu} & \text{if } V^S_{a,n} > 0 \end{cases}. \tag{11}$$

For low production technologies n, private firms will never find IPOs beneficial because the public firm value does not offset the fixed IPO costs. At higher production technologies, where $\mathbb{E}_v V^{Pri}_{a,n}(v) > V^S_{a,n} > 0$, some firms may prefer IPOs because of preferences, ε, even though public firms are not strictly more valuable. While at high-enough production technologies, where $V^S_{a,n} > \mathbb{E}_v V^{Pri}_{a,n}(v)$, firms will tends to prefer IPOs, although some may remain private due to preferences. This could capture, for example, the preference of keeping control of a family-owned firm within the family.

The entry problem is given by:

$$V_0 = \max_u u[\omega_{a_l} \tilde{V}^{Pri}_{a_l,1} + \omega_{a_h} \tilde{V}^{Pri}_{a_h,1}] - \psi_E u^\zeta, \tag{12}$$

where the first term is the expected value of a newly created private firm is and the second term is the entry cost.

3.3.3 Firm Distribution

The evolution of the share of type (κ, a, n) firms is given by:

$$\Psi^{\kappa}_{a,n}{}' = \begin{cases} Entry^\kappa_a + (1-\nu)\left[(1 - x^\kappa_{a,1} - \chi)\Psi^\kappa_{a,1} + \chi\Psi^\kappa_{a,2}\right] + S^\kappa_{a,1} & \text{if } n = 1 \\ (1-\nu)\left[(1 - x^\kappa_{a,n} - \chi)\Psi^\kappa_{a,n} + \chi\Psi^\kappa_{a,n+1} + x^\kappa_{a,n-1}\Psi^\kappa_{a,n-1}\right] + S^\kappa_{a,n} & \text{if } n > 1 \end{cases}. \tag{13}$$

The entry rate $Entry^\kappa_a$ is the share of new entrants, which equals the average exit rate multiplied by the entry rate of type a firms, $\omega_a \sum_\kappa [\chi(1 - \nu^\kappa_n)\Psi^\kappa_{a,1} + \sum_n \nu\Psi^\kappa_{a,n}]$ for private firms and zero otherwise. Finally, the IPO share $S^{\kappa}_{a,n}$ is the share of new public firms that enter on node n and equals $S^{Pub}_{a,1} = s_{a,1}[Entry^{\kappa} + (1-\nu)[(1-x^{Pri}_{a,1}-\chi)\Psi^{Pri}_{a,1} + \chi\Psi^{Pri}_{a,2}]]$ for $n = 1$ and $S^{Pub}_{a,n} = s_{a,n}(1-\nu)\left[(1-x^{Pri}_{a,n}-\chi)\Psi^{Pri}_{a,n} + \chi\Psi^{Pri}_{a,n+1} + x^{Pri}_{a,n-1}\Psi^{Pri}_{a,n-1}\right]$ for $n > 1$ and $S^{Pri}_{a,n} = -S^{Pub}_{a,n}$. The stationary distribution follows from setting ${\Psi^{\kappa}_{a,n}}' = \Psi^{\kappa}_{a,n}$. It follows from the above definition that $\sum_{\kappa,a,n}\Psi^{\kappa}_{a,n} = 1$.

The mass of firms evolves according to:

$$M' - M = 0 = u - \sum_{\kappa,a}\left[\left(\chi(1-\nu)\Psi^{\kappa}_{a,1} + \sum_{n}\nu\Psi^{\kappa}_{a,n}\right)\right]M. \tag{14}$$

The stationary mass sets the entry rate (first term) equal to the exit rate of firms (second term). The exit rate depends on the distribution of firms $\Psi^{\kappa}_{a,n}$ across corporate structures κ, types a, and technologies n, which also determine the relative probability of exiting.

3.3.4 Market Clearing

Labor and capital markets. The input markets require that the resources in the economy are employed by firms and follow from the labor and capital demand in (6) and (7). Labor market clearing requires that

$$1 = M\left[\left(\frac{\gamma(1-\alpha)}{w}\right)\left(\frac{\alpha}{(1-\alpha)}\frac{w}{r}\right)^{\alpha\gamma}\right]^{1/(1-\gamma)}\sum_{\kappa,a,n}\int_{v}\tau^{\kappa}_{a,n}(v)^{-\frac{\alpha\gamma}{1-\gamma}}z^{\kappa}_{a,n}(v)d\Phi(v)\Psi^{\kappa}_{a,n}, \tag{15}$$

where the left-hand side is the supply of workers and the right-hand side is the demand for labor by all firms. Capital market clearing requires that

$$K = M\left[\left(\frac{\gamma\alpha}{r}\right)\left(\frac{(1-\alpha)}{\alpha}\frac{r}{w}\right)^{\gamma(1-\alpha)}\right]^{1/(1-\gamma)}\sum_{\kappa,a,n}\int_{v}\tau^{\kappa}_{a,n}(v)^{-\frac{1-(1-\alpha)\gamma}{1-\gamma}}z^{\kappa}_{a,n}(v)d\Phi(v)\Psi^{\kappa}_{a,n}. \tag{16}$$

Both the labor and capital market clearing depends on a capital-wedge weighted average productivity of firms that captures the impact of the capital wedge τ^{κ} on firm-level demand for labor and capital.

Aggregate output. Aggregate output depends on the joint distribution of firm productivities, corporate structure, and capital wedges. Aggregate output is given by:

$$Y = AM^{1-\gamma}(K^{\alpha}L^{1-\alpha})^{\gamma}. \tag{17}$$

where

$$A = \frac{\sum_{\kappa,a,n} \int_v \tau^{\kappa}_{a,n}(v)^{-\frac{(1-\alpha)\gamma}{1-\gamma}} z^{\kappa}_{a,n}(v) d\Phi(v) \Psi^{\kappa}_{a,n}}{\left(\sum_{\kappa,a,n} \int_v \tau^{\kappa}_{a,n}(v)^{-\frac{\alpha\gamma}{1-\gamma}} z^{\kappa}_{a,n}(v) d\Phi(v) \Psi^{\kappa}_{a,n}\right)^{(1-\alpha)\gamma} \left(\sum_{\kappa,a,n} \int_v \tau^{\kappa}_{a,n}(v)^{-\frac{1-(1-\alpha)\gamma}{1-\gamma}} z^{\kappa}_{a,n}(v) d\Phi(v) \Psi^{\kappa}_{a,n}\right)^{\alpha\gamma}} \tag{18}$$

The aggregate economy nests a standard neoclassical growth model with productivity A. The economy has constant returns to scale in terms of capital K, labor L, and the mass of firms M (which is proportional to the Cobb-Douglas aggregate of capital and labor), where the coefficient on the mass of firms $1-\gamma$ captures the impact of decreasing returns to scale on production. Aggregate productivity depends on two factors that are important for the quantitative analysis. First, aggregate productivity depends on capital misallocation driven by the capital wedge $\tau^{\kappa}_{a,n}(v)$. In the case where there is no capital misallocation, $\tau^{Pri}_{a,n}(v) = \tau^{Pub}_{a,n}(v) = 1$, then the expression for aggregate productivity simplifies to the average productivity of firms to the exponent $1-\gamma$, the same exponent in the firm-level production function. Second, aggregate productivity depends on the endogenous productivity distribution $\Psi^{\kappa}_{a,n}$. The distribution depends on both the endogenous selection of firms into corporate structures κ as well as the investment in improving technologies n, which is also affected by corporate structure.

Output is used for consumption, IPO costs, investment in production technologies, and the capital stock. The goods market clearing condition is then given by

$$Y = C + M \sum_{\kappa,a,n} \int_v x^{\kappa}_{a,n}(v) d\Phi(v) \Psi^{\kappa}_{a,n} + F + \delta K \tag{19}$$

where IPO costs are $F = M \sum_{a,n} [(1-\nu) x^{Pri}_{a,n-1} \Psi^{Pri}_{a,n-1} + (1-\nu)(1 - x^{Pri}_{a,n} - \chi) \Psi^{Pri}_{a,n}] s_{a,n} f$ and output Y is defined in (17) and (18). IPO decisions occur in the middle of the period (step iv in the timing description) and so IPO costs depend on a distribution that differs from the $\Psi^{\kappa}_{a,n}$ described above, after the realization of exit, obsolescence, and technology improvements shocks but before IPO decisions.

3.3.5 Equilibrium Definition

A stationary Markov Perfect Equilibrium consists of the values

$$\{C, K, T, \ell^{\kappa}_{a,n}(v), k^{\kappa}_{a,n}(v), x^{\kappa}_{a,n}(v), d_{a,n}(\varepsilon), s_{a,n}, u, \Psi^{\kappa}_{a,n}, M, r, R, w\}$$

for $\kappa \in \{Pri, Pub\}$, $n = \{0, 1, 2, ..., N\}$, $\varepsilon \geq 0$, and $v \in \mathbb{R}$, such that:

1. Households maximize utility in (4);
2. Firms choose (ℓ, k) to maximize profits according to (6) and (7);
3. Firms choose $x^{\kappa}_{a,n}(v)$ to maximize value in (8) and (9);
4. The IPO decision $d_{a,n}(\varepsilon)$ and IPO probability $s_{a,n}$ are given by (10) and (11);
5. Potential entrants set u maximize expected value in (12);
6. The distribution $\Psi^{\kappa}_{a,n}$ and mass of firms M are stationary and given by (13) and (14);
7. The government's budget is balanced by T;
8. The labor market (15), capital market (16), and goods market (19) clear.

4 Model Calibration

We use the calibrated model to examine the contribution of public equity markets to aggregate productivity in France, Germany, Spain, and Italy. We calibrate each economy to data moments on firm dynamics, the firm distribution, and public-private firm differences. For brevity, we discuss the model calibration and goodness of fit for the French data and end the section by presenting the parameter estimates for all four economies. The fit of the other three economies is provided in Appendix C.2.

4.1 Calibration Strategy

The calibrations sets the parameters $\{\theta, \beta, \delta, \gamma, \alpha, \omega_{a_l}, a_l, \psi_E, \psi, \zeta, \lambda, \eta, \sigma_v, \phi, \nu, \mu, f, \bar{\tau}^{\kappa}, \rho^{\kappa}\}$. We normalize the productivity of high-type firms to $a_h = 1$. For computational purposes, we set the number of technologies $n = \{0, 1, 2, ..., N\}$ to $N = 50$ and idiosyncratic productivities v to five nodes on the grid $\{-2\sigma_v, -\sigma_v, 0, \sigma_v, 2\sigma_v\}$.

Externally set parameters. The discount rate is set to $\beta = 0.96$ and the coefficient of relative risk aversion is set to $\theta = 2/3$. The span-of-control parameter to $\gamma = 0.85$ (as in, for example, Restuccia and Rogerson, 2008) and the capital share to $\alpha = 0.33$. The curvature on investment is set to $\zeta = 2$ as in Acemoglu et al. (2018). We normalize the entry cost $\psi_E = 1$ since this only affects the equilibrium firm mass M.

We set the taste parameter on private and public firms to $\mu = 50$ such that firms are highly value sensitive when making IPO decisions. We find that this choice of μ leads to a similar share of large firms (greater than 250 employees) as in the data. We hold this parameter fixed in the calibration of economies to other countries.[4] We set the investment scaling factor to $\phi^{\kappa} = 1 - \rho^{\kappa}\alpha\gamma/(1-\gamma)$ to offset the impact of the capital wedge elasticity on the change in profits as firms move to higher production technologies causing investment costs to scale with firm profitability at higher production technologies. In the absence of other factors (i.e., public equity markets, obsolescence shocks), this assumption would imply that Gibrat's law holds in equilibrium, which is a useful benchmark.

Estimated capital wedges. We directly estimate the capital wedges $\tau^{\kappa}_{j,t}$. Following the model, the capital wedge is proportional to $Wedge^{k}_{j,t} = \tau^{\kappa}_{j,t} \propto y_{j,t}/k_{j,t}$ allowing us to construct the capital wedge directly using data.[5] We estimate the parameters $\bar{\tau}^{\kappa}$ and ρ^{κ} by regressing the capital wedge on firm TFP allowing for differences in the intercept and slope of public and private firms. The results are given by:

$$\ln Wedge^{k}_{j,t} = \underset{(0.000664)}{1.325} \ln TFP_{j,t} + \left[\underset{(0.204)}{2.873} - \underset{(0.0200)}{0.525} \ln TFP_{j,t}\right] Pub_{j,t} + \Gamma_s + \Gamma_t + u_{j,t}, \quad (20)$$

where standard errors are reported in parentheses and $Pub_{j,t}$ is a variable that takes value one if firm j is public in period t. We include sector and year fixed effects Γ_s and Γ_t to avoid capturing between sector or year variation in capital stocks.

The results show that the intercept for public firms is substantially higher than for private firms. This is traded off by a lower slope of capital wedges with respect to firm-level TFP implying that productive firms face lower capital wedges as public firms. An issue with relating the expression directly with capital wedges in the model is that TFP does not have a natural base. To deal with this, we include a reference TFP ($\overline{TFP}$) in the joint parameter calibration that converts the estimated capital wedges into model values. In the calibrated model, the lower slope coefficient of public firms dominates the higher intercept at productivities where firms go through IPOs indicating that public firms face lower capital wedges, consistent with the evidence in section 2.3 and appendix A.3.

A potential concern with the above results is that less distorted firms may select into IPOs. We find two facts that suggest that this is not the main driver of the above relationship. First, when we look at the assets of individual firms, we see large increases around the time of the firm IPOs (Section 2.3) beyond the corresponding increases in employment or output, indicating that some of the gain is coming through a decline in the capital wedge. For this reason, we use the change in firm-level assets around the IPO as a moment in the calibration to discipline the reference productivity to relate the regression in (20) to the model. Second,

[4] The inclusion of μ also avoids kinks in the labor demand function simplifying the computational problem.

[5] This is the same as the estimation of MRPK in Hsieh and Klenow (2009).

when we regress labor wedges on firm-level TFP and a public indicator, we see much smaller differences between public and private firms (Appendix A.3) indicating that these differences are coming through the capital market.

Jointly calibrated parameters. The parameters $\{\nu, \chi, \psi, \lambda, \sigma_v, a_l, \omega_{a_l}, \eta, f, \overline{TFP}\}$ are jointly calibrated to match the data moments in Table 5. The exit rate and exit rate elasticity relate to the exogenous exit rate ν and obsolescence shock χ. The innovation parameters ψ and λ and the dispersion of random shocks σ_v relate to the level and dispersion of employment and employment growth. The low-type firm parameters a_l and ω_{a_l} relate to the firm and employment share of small firms. The IPO cost f and the residual public productivity component η relate to the employment share of public firms and the relative size of public firms. Finally, we use the change in the firm's capital around its IPO to determine the reference productivity $\overline{TFP}$ for the capital wedges. The construction of the moments in the data and model are discussed in more detail next.

Table 5: Model and Data Moments

Moment	Data	Model
Entry Rate (%)	10.7	10.6
Exit Rate Elasticity	-0.0005	-0.00052
Avg Emp Growth (%)	4.14	4.24
Std Emp Growth	27.2	26.9
Std Employment	0.76	1.16
Firm Share of Small Firms (%)	84.5	85.0
Emp Share of Small Firms (%)	24.3	28.1
Emp Share of Public Firms (%)	23.0	23.8
Relative Size of Public Firms	219.7	230.3
IPO Capital Growth	0.9	1.4

4.2 Data and Model Moments

The jointly calibrated parameters are chosen to minimize the sum of absolute errors between the model and data moments.[6] While these moments are highly inter-dependent, we discuss the construction of the moments and closely related parameters below. Appendix C.3 provides additional details of the calibration. All of the moments calculated using firm-level data are adjusted for firm weights as described in Section 2.1.

[6]Specifically, we minimize $\sum_i |\mathcal{M}_i^{Model} - \mathcal{M}_i^{Data}| / (0.5|\mathcal{M}_i^{Model} + \mathcal{M}_i^{Data}|)$ as in Akcigit and Kerr (2018), where $\mathcal{M}_i^{Model}$ and $\mathcal{M}_i^{Data}$ are model and data moments.

Entry Rate. The model moment is constructed as new firms u divided by the mass of firms M, which is equivalent to the exit rate since we examine the stationary equilibrium. The data moment is constructed as total new firms divided by the total number of firms in the Eurostat data. The entry rate disciplines the exogenous exit rate ν and the obsolescence shock χ.

Exit Rate Elasticity. The obsolescence shock χ implies that small firms are more likely to exit than large firms. We discipline this moment using the elasticity of firm exit with respect to firm size. In the model, we construct this moment as the coefficient from regressing the probability of firm exit on the log of firm size. In the data, we construct the moment using Eurostat data by regressing the average exit rate of firms in a size bin on the log of the average size of firms in the size bins.[7]

Avg Emp Growth. We calculate the growth rate of firm j as $g_{j,t+1} = (\ell_{j,t+1} - \ell_{j,t})/(0.5 * (\ell_{j,t+1} + \ell_{j,t}))$ in the model and the data, where we construct growth in this manner to minimize the influence of outliers in the data. The model moment is calculated as the expected value of $g_{j,t+1}$ given the stationary firm distribution $\Psi^{\kappa}_{a,n}$ and idiosyncratic shock distribution $\Phi(v)$. The data moment is calculated over all firms with data in consecutive periods, where we use weights in the latter period. The average employment growth helps discipline both the average step size of firm growth λ and investment cost ψ.

Std Emp Growth. The standard deviation of employment growth is calculated using the previously described growth rates $g_{j,t}$. The standard deviation of employment growth helps discipline both the step size λ, the cost of investment ψ, and the idiosyncratic productivity dispersion σ_v. For a given average growth rate, less frequent but larger innovations will result in larger deviations of employment growth.

Std Employment. The model moment is calculated using the implied standard deviation of log employment based on the stationary distribution of firm types. The data moment is calculated as the average across periods of the standard deviation of log employment. The moment is closely related to the parameters that determine the relative dispersion in productivity, such as the cost of investment technology (ψ, λ), the low-type productivity a_l and probability ω_{a_l}, and the dispersion σ_v in idiosyncratic productivity.

[7] Specifically, we target the coefficient β_1 from the regression $\log(\overline{Exit}_{b,t}) = \beta_0 + \beta_1 \times \log(\overline{Emp}_{b,t}) + \Gamma_t + e_{b,t}$ for size bins b and year t, where Γ_t is a year fixed effect.

Emp and Firm Share of Small Firms. We define small firms as firms employing fewer than ten workers in both the data and the model. To map employment in the model to the data, we assume that the typical low-type private firm with idiosyncratic productivity $v_{j,t} = 1$ employ a single worker, i.e., $\ell^{Pri}_{a_\ell,1}(1) = 1$. We then calculate the model moments as the share of firms and employment by firms with fewer than nine times this level of employment. We take the data moment from the Eurostat data. The moments help discipline the relative frequency ω_{a_l} and size a_l of low-type firms but is also related to firm production technology improvements through the investment cost ψ and step size λ.

Emp Share of Public Firms. We construct the empirical public firm employment share using a variety of data sources. We compute the total number of employees in public firms from Compustat global. We then divide this by the number of non-government employees in a country using data from the OCED and the ILO. See appendix A.5 for full details. We construct the model moment using the public firm and employment share using the stationary distribution across firm types. The moment is closely related to the fixed IPO cost f and the residual productivity of public firms η. The moment also help to discipline characteristics of large firms in the model and, consequently, are closely related to the investment technology (ψ, λ) and the obsolescence shocks χ since these parameters all discipline the relative share and productivity of large firms.

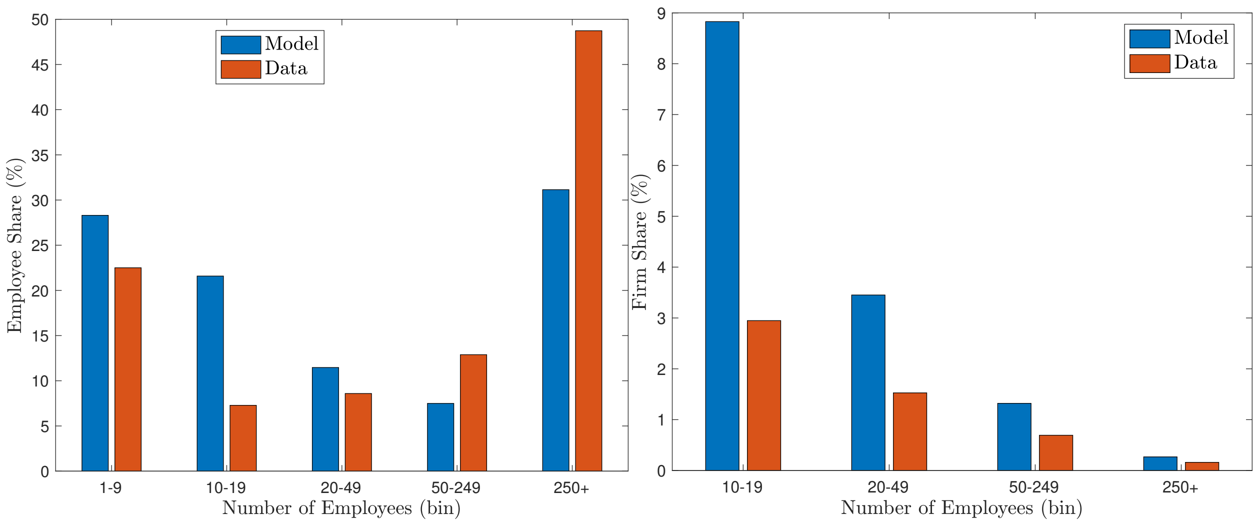
Relative Size of Public Firms. We construct the model moments as the average employment of public firms divided by the average employment of private firms. We construct the data moment similarly using the data reported in Table 1. The moment helps discipline the fixed IPO costs and the residual public-specific productivity component η.

IPO Capital Growth. We calculate the moment as the expected log change in capital of firms between the year prior to an IPO and the year following an IPO. We calculate the model moment using the stationary IPO distribution implied by the stationary firm distribution $\Psi^\kappa_{a,n}$ and idiosyncratic distribution $\Phi(v)$. The data moment is calculated using the coefficients estimated in Section 2.3. The moment helps discipline the reference productivity $\overline{TFP}$ used to relate capital wedges estimated in the data to the model and is also closely related to other parameters related to public-private differences (i.e., η and f).

4.3 Goodness-of-Fit and Other Moments

Figure 3 compares the firm and employment distributions over employment size bins in the model and empirical data. For the firm distribution, we omit the lowest size bin since it contains the majority of observations and distorts the figure's scale.

Figure 3: Employment and Firm Distribution (France)



(a) Employment Distribution

(b) Firm Size Distribution

Notes: Figure (a) reports the share of employment by firms in different employment bins. Figure (b) reports the share of firms in different employment bins. The model moment is calculated using the stationary distribution of firm types and the assumption that the typical low-type private firms employs a single worker, $\ell_{a_l,1}^{Pri} = 1$. *The data moment is taken from the Structural Business Statistics database from Eurostat for 2015.*

The model does well at replicating the shape of the firm and employment distributions, despite only the smallest bin being directly targeted in the calibration. This is, in part, due to public firms acting as a stand in, albeit imperfect, for large firms in the calibration. At the top of the distribution, the model slightly over-predicts the share of large firms but under-predicts the employment share of large firms. This is likely in part due to the quantitative model limiting the upper bound of firm employment.[8]

Table 6 reports other model and data moments that are not directly targeted in the calibration. These serve as a goodness-of-fit check. The table reports the relative assets, output, and TFP between public and private firms, the standard deviation and growth rates of assets, output, and TFP and the IPO employment growth.

Overall, the calibrated model fits the data well. The model implied relative output and TFP between public and private firms match closely to the data. The model underpredicts the relative assets between public and private firms. However, we find that this is, at least in part, driven by sectoral differences between public and private firms.[9] The model also

[8]The largest firm in the calibrated model has over 2,000 employees whereas the largest French firms employ over 100 thousand employees.

[9]We regress log assets on a public dummy including sector-by-year fixed effects. The implied level of

Table 6: Other Moments

Moment	Data	Model
Relative Assets	3429	139.9
Relative Output	341.5	230.3
Relative TFP	2.98	3.13
Std Assets	1.83	1.0
Std Output	1.0	1.2
Std TFP	0.67	0.2
Avg Asset Growth (%)	2.21	2.9
Avg Output Growth (%)	5.25	4.2
Avg TFP Growth (%)	2.21	1.02
IPO Employment Growth	0.44	0.8

matches the standard deviation and growth of assets and output The model understates the growth and standard deviation of TFP, which is driven in part by the relationship between TFP and employment not directly being targeted in the calibration.[10]

4.4 Calibration of Other Countries

Table 7 presents the calibration parameters for France as well as the other countries used in the analysis. The parameters for different countries tend to be in similar ranges. Comparing the parameters highlights a few important trends across countries.

Table 7: Comparison of Country Parameters

Parameter		FR	DE	IT	ES
Exit Rate (%)	ν	10.6	8.1	7.1	9.0
Productivity Step	λ	1.20	1.22	1.19	1.19
Investment Cost	ψ	0.04	0.10	0.07	0.05
Obsolescence Shock (%)	χ	0.14	0.59	0.37	0.46
Public Productivity	η	0.51	0.53	0.32	0.50
Low-Type Productivity	a_l	0.27	0.16	1.07	0.46
Low-Type Probability (%)	ω_{a_l}	47.3	1.7	17.4	53.6
Idiosyncratic Dispersion	σ_v	0.24	0.20	0.35	0.28
IPO Cost	f	20.0	2.2	269.3	140.4
Reference Productivity	$\overline{TFP}$	5.5	7.1	13.7	3.7

relative assets from this regression is around 300 times, much closer to the model relative assets in Table 6.

[10] We find a positive correlation between labor wedges and TFP in the data. Disciplining the calibrated model to labor growth and dispersion implies—in the absence of labor wedges—lower dispersion in TFP, consistent with Table 6.

The calibration of the French benchmark economy implies that low-type firms are around 18% ($\approx 1 - a_l^{1-\gamma}$) less productive than high-type firms. Firms lose just under 10% ($\approx 1 - \eta^{1-\gamma}$) of their productivity when they go from private to public. The calibrated value of the residual productivity term η is consistent with agency frictions in public firms with diluted management stakes, as discussed in Peter (2021). While the decline in productivity is substantial, this is more than outweighed by the increase in capital associated with lower capital wedges, implying that a substantial fixed IPO cost (see discussion below) is still necessary to limit IPOs.

Table 8: Implied IPO Costs

	FR	DE	IT	ES
IPO Cost in Thousands 2021 USD	288	206	2,444	1,111

Notes: Values reported in thousands of 2021 US dollars. Implied IPO cost is calculated as f divided by aggregate output Y multiplied by the country's GDP per capita in 2021 USD.

The fixed IPO cost f varies substantially between France (deepest equity market) and Italy (shallowest equity market). The measured fixed IPO cost is an order of magnitude larger in Italy compared with France. On its own, the fixed IPO cost reflects large differences in the ease of conducting IPOs across countries. However, it is also important to note that other factors will lead to differences in the public share of firms across countries. For example, differences in the exit rates ν imply that firms may be more or less likely to survive to being large enough to go through IPOs while differences in the share of high-type firms $(1 - \omega_{a_l})$ imply differences in the potential pool of firms that can go through IPOs. This highlights the importance of jointly calibrated the model parameters as cross-country differences in the firm distributions can affect the necessary fixed IPO costs f in order to match the equity market depth observed in different countries. An upshot of these other factors is that Germany, which has a shallower equity market than France in the data, has a lower fixed IPO cost than France in the calibrated model. Germany can have a lower fixed IPO cost and less public employment because there is less benefit from firms going public in the German economy.

Table 8 reports the fixed IPO cost f reports in equivalent 2021 USD values for each country. For this conversion, we divide the fixed IPO cost f by model output (per capita) Y and multiply by the GDP per capita of each economy in 2021 USD. This conversion allows for an easier interpretation than the model value of f. The values range from around 200 thousand USD in Germany to around 2.4 million USD in Italy. The cross-country comparison of values in Table 8 is largely the same as the calibrated values of f.

5 Cross-Country Importance of Public Equity Markets

In Section 2.1, we show that public firms tend to be substantially larger, more productive, and better capitalized than private firms in each of the four economies. In this section, we use the quantitative model to provide estimates of the value and contribution of public equity markets to aggregate productivity and output.

5.1 Value of Public Equity Markets

We start by estimating the value of public equity markets to each of the four calibrated economies. We measure the value of public equity markets as the change in aggregate output between economies with and without public equity markets. The latter economies are identical to the calibrated economies in Table 7 but with fixed IPO costs set prohibitively high (i.e., $f \to \infty$) such that no firm conducts an IPO. Table 9 reports the estimated values from the experiment.

Table 9: Value of Equity Markets

Parameter	FR	DE	IT	ES
Output (%)	5.9	1.5	0.5	1.3
TFP (%)	7.7	1.2	-0.4	1.2
Mass of Firms (%)	0.1	0.1	-0.0	0.0
Investment / GDP (pp)	0.5	0.2	-0.0	0.0
Firm Share of Large Firms (pp)	0.2	0.1	-0.0	0.0
Emp Share of Large Firms (pp)	21.3	5.8	2.3	5.8

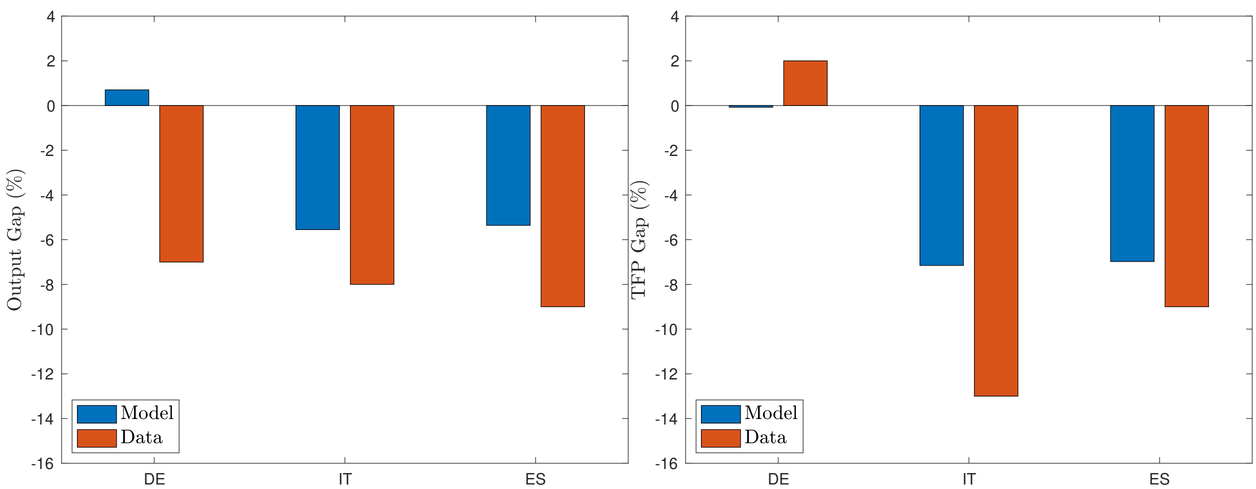
Notes: The figure reports the percent change (%) and percentage point change (pp) of moving from the counterfactual economy (with $f \to \infty$) to the benchmark economy for each indicated country. Parameter values in the benchmark economy are reported in Table 7. Large firms are firms that employ more than 250 employees.

The value of public equity markets ranges from around 0.5% of GDP in the shallowest market (Italy) to around 6% of GDP in the deepest market (France). In the French benchmark economy, public firms account for around 24% of total output, substantially more than the 6% reported in Table 9. Part of the difference in these values has a simple explanation. Some share of large public firms in the benchmark economy become large private firms in the counterfactual economy. While the contribution of these firms may be lower in the counterfactual economy, due to higher capital wedges, they are not removed from the economy and their overall contribution to aggregate productivity remains high. The 6% of GDP for France then measures the share of additional large firms (from higher investment), the higher mass of firms, and the improvement in the capital allocation that result from public equity markets.

5.2 Cross-Country Productivity Differences

Next, we use the model to understand the contribution of cross-country differences in public equity market development, captured by the fixed IPO cost f, on aggregate productivity. The calibration of individual countries in Section 4.4 highlights how differences in equity market depth translate into model differences in fixed IPO costs f. The fixed IPO cost captures both technological gaps (e.g., infrastructure, digitalization of financial market) and institutional barriers (e.g., institutional trust, culture) that lead to differences in IPO costs across countries. Given that some technological differences across countries may not be resolved through financial development, we consider changes in relative IPO costs across countries. This allows us to make cross-country comparisons without taking a stance on how much of the fixed IPO cost f could potentially be lowered through financial development.

Figure 4: Cross-Country Equity Markets and Productivity



(a) Output (b) TFP

Notes: The figures report the percent different in output per capita (Panel a) and TFP (Panel b) between the counterfactual and benchmark economies for the model and between the indicated country and France for the data. Counterfactual economies adjust the fixed IPO cost in the French benchmark economy to match the estimated value for the indicated country. TFP is calculated as aggregate output divided by the Cobb-Douglas aggregates of aggregate labor and capital.

We measure the aggregate importance of public equity markets through a counterfactual exercise. We start with the economy calibrated to French data. We then adjust the fixed IPO cost f to match the values estimated in Germany (DE), Italy (IT) and Spain (ES). Figure 4 reports the results. The model moments are calculated as the percent change in aggregate output and TFP in the counterfactual economy relative to the baseline economy. The data moments are calculated as the percent difference in aggregate output per capita and TFP between France and Germany, Italy, and Spain — i.e., a 8% output gap in the data indicates that Italy produces 8% less output per capita than France.

The experiment shows that public equity markets differences vary quite substantially across countries and explain a large share of productivity differences for some countries. The comparison of France to Germany shows that equity market differences do not play a substantial role. Despite the relatively shallower public equity markets in Germany, the measured fixed IPO cost f are lower than in France, although the quantitative differences are small. Through the lens of the model, this implies a relatively small productivity change from moving to the German fixed IPO costs. The comparisons of France to Italy and Spain show a more important role for public equity markets. Differences in the fixed IPO costs f explain between half and three quarters of the output per capita and TFP gaps in the data.

Table 10: Counterfactual Economy Differences

Parameter	DE	IT	ES
Mass of Firms (%)	0.1	-0.1	-0.1
Investment / GDP (pp)	0.2	-0.5	-0.5
Firm Share of Public Firms (pp)	0.3	-0.1	-0.1
Emp Share of Public Firms (pp)	7.1	-23.8	-21.4
Relative Size of Public Firms (%)	-51.3	-	51.7

Notes: The table reports the percent change (%) and percentage point change (pp) in outcomes between the counterfactual and benchmark economies. Counterfactual economies adjust the fixed IPO cost in the French benchmark economy to match the estimated value for the indicated country.

Table 10 sheds some light on the drivers of the aggregate productivity differences by showing the change in outcomes related to the firm investment and the overall importance of public firms in the economy. The table shows that the relative importance of public firms increases in Germany, where fixed IPO costs are lower, and decreases for Italy and Spain, where fixed IPO costs are higher. In the Italy and Spain counterfactuals, there is less investment by incumbent and entrant firm, reflecting the incentivizing effect of IPOs on firm investment. The relative size of public firms declines in Germany as the lower fixed IPO cost allows smaller firms to become public on the margin, while the reverse happens in Spain. The Italian fixed IPO cost is so large that no firm goes public in the counterfactual economy.

5.3 Importance of Private and Public Differences

The third experiment we consider examines how within-country differences between private and public firms affect aggregate productivity. The primary difference between private and public firms is in their access to capital markets, which is parameterized by differences in the level $\bar{\tau}^{\kappa}$ and elasticity ρ^{κ} of capital wedges. The elasticity of capital wedges determines the relative distance of more productive firms from the first-best allocation, i.e., the extent of capital misallocation.

We focus on differences in the the elasticity of capital wedges between public and private firms, given the general importance of this parameter in the misallocation literature (e.g., Restuccia and Rogerson, 2008).[11] We consider an experiment in which the elasticity of capital wedges ρ^{κ} is equated between private and public firms by setting ρ^{Pri} to ρ^{Pub}. Table 11 reports the results. Except for in Italy, the counterfactual economies have a flatter profile of capital wedges with respect to firm productivity and firms in the counterfactual economies choose not to go public, since there is no incremental benefits. In Italy, the estimated elasticity is higher for public firms causing it to be an outlier compared with the other countries. We present the results for Italy for completeness, but focus on the other countries. Because our goal is to look at the impact of capital wedges, we also adjust the investment scale parameter ϕ such that investment costs scale proportionally to profits, as in the benchmark economies.

Table 11: Gains from Improving Capital Markets

Parameter	FR	DE	IT	ES
Output (%)	26.7	21.8	-4.0	34.6
TFP (%)	21.9	18.5	-5.8	28.6
Mass of Firms (%)	7.0	15.2	-1.9	6.1
Investment / GDP (pp)	3.6	2.8	-1.0	4.8
Firm Share of Large Firms (pp)	1.5	1.0	-0.1	1.6
Emp Share of Large Firms (pp)	32.1	26.3	7.5	45.1

Notes: The table reports the percent change (%) and percentage point change (pp) in outcomes between the counterfactual and benchmark economies. Counterfactual economies sets the elasticity of capital wedges ρ^{κ} of private firms to match public firms, i.e., $\rho^{Pri} = \rho^{Pub}$.

The table highlights that aggregate output per capita and TFP increase by between

[11] We could also change the level $\bar{\tau}^{\kappa}$ but since this parameter is common for all firms in the counterfactual economy (since firms no longer go through IPOs), this does not impact the allocative efficiency of capital. It would result in a level shift of profits that could affect investment, but it is unclear how this should be interpreted.

19% and 35% relative to the benchmark economies, except in Italy where both decline. The value of lowering the capital wedge elasticities is that it improves the allocative efficiency of capital, since more productive firms rent more capital, and the higher incremental profits incentivize incumbent and entrant investment. Together, these channels result in higher average firm productivity and economic activity (i.e., output, employment, hiring capital) being reallocated to relatively productive firms.

Another difference between public and private firms is the public component of productivity η. The calibrated value of η is less than one in all four economies indicating that firms loss some productivity when they go public. Appendix C.4 reports the aggregate implications of setting $\eta = 1$. We find comparable increases in aggregate productivity as in Table 11 for this experiment for France, Germany, and Spain, and large positive gains for Italy, with the smallest calibrated η.

6 Policy Experiment

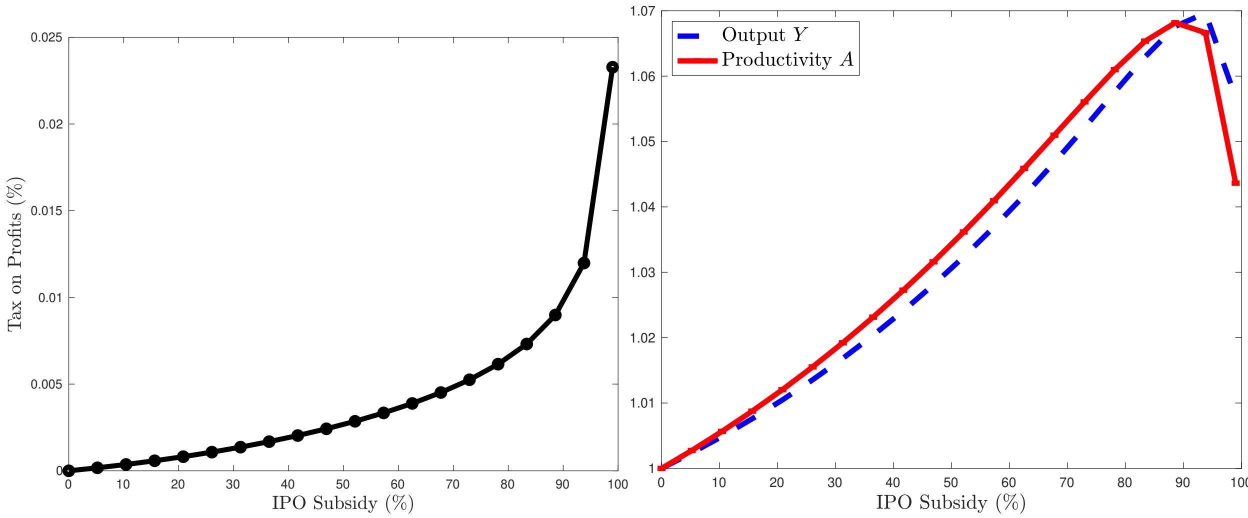
Finally, we use the quantitative model to enact a simple policy designed to encourage more firms to become public. We consider a subsidy s to the fixed IPO cost f such that the new fixed IPO cost becomes $(1-s)f$. The subsidy is paid for with a tax on firm profits τ^π such that firm j's profits become $(1-\tau^\pi)\pi_j$, for all j. The experiment sets the tax on profits such that the government's budget is balanced for a given subsidy, i.e., such that $\int_j d_j f s dj = \int_j \tau^\pi \pi_j dj$. Increasing the subsidy to the fixed IPO cost results in more public firms, since the necessary threshold to go public is lower, and increases the incentives of private firms to invest near the IPO production technology, since the incremental benefit from going public is higher. However, the cost of the policy is a higher tax on firm profits which disincentivizes investment, especially at lower production technologies where private firms weigh the future benefits of going public less. While we model the cost as a tax in profits, the underlying economics would be similar if this were instead a tax on labor or capital, since these taxes would similarly depress profits.[12]

Figure 5 reports the results from the experiment using the French benchmark economy. Panel (a) summarizes the necessary tax at different levels of subsidies and Panel (b) summarizes the impact on aggregate output and productivity. Aggregate output and productivity are normalized to one in the benchmark economies, with zero subsidy.

The first takeaway from the experiment is that subsidies to equity market depth can

[12] In our model, a tax on labor would be canceled out by a general equilibrium adjustment in the wage rate because of the fixed labor supply. However, it is straightforward to consider an extension to the labor market where households make aggregate labor supply decisions and this is not the case.

Figure 5: Policy Experiment (France)



(a) Profit tax rate

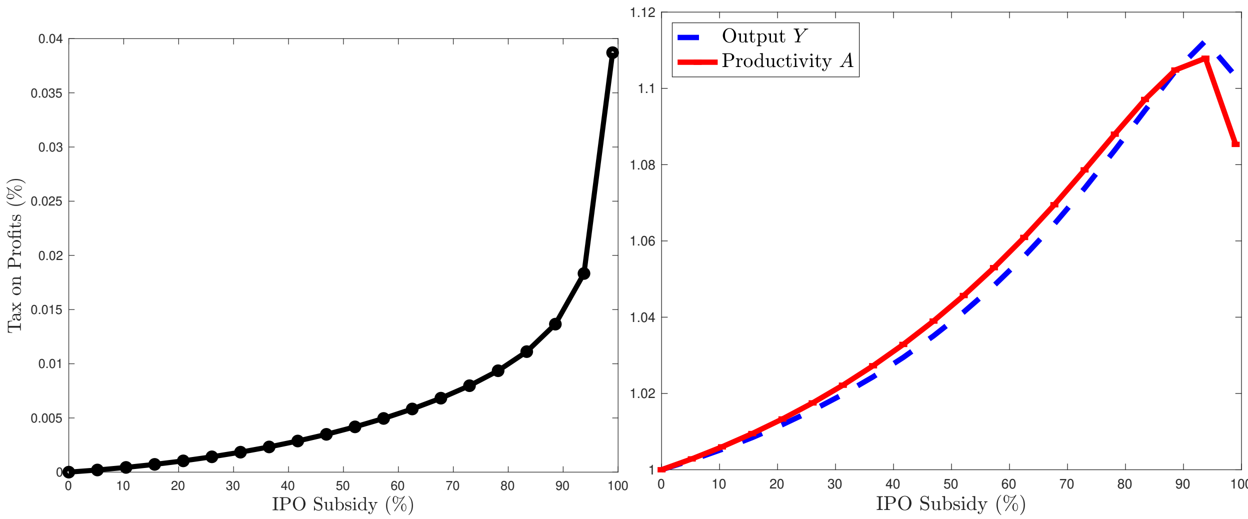
(b) Output and Productivity

result in large gains to aggregate productivity, even when funded by a budget neutral tax. Intuitively, public firms account for a relatively small share of firms, only around 0.4% of firms in the benchmark economy, but a large share of employment, around 40% in the benchmark economy. This implies that the costs to subsidizing public firms is paid only to a small number of firms but benefits a large share of the workforce. This is further amplified since public firms only pay the fixed IPO cost at one point in time but the benefits from public firms are received each period and even to firms prior to going public, through the incentivizing effect of IPOs on firm investment. Similarly, the negative impact of the financing profit tax is spread over a relatively large base and so only needs to have minimal distortions on the overall economy.

The second takeaway is that at large subsidies, the marginal benefits from further subsidization of IPO costs can turn negative. On one hand, as the IPO subsidy increases it requires a larger tax on firm profits to finance, putting further pressure of incentives to invest in productivity improvements. On the other hand, as the subsidy increase the marginal firm that goes public become lower productivity and has a smaller incremental benefit from accessing public capital markets. This leads the hump-shaped increase in output and productivity.

Figure 6 reports the same experiment using the Spanish benchmark economy rather than the French benchmark economy. The quantitative impact of the results are stronger, gains in productivity peak at around 11% in Spain compared with 7% in France, but display a similar pattern as in the French economy. The similarities of the results are consistent with the similar parameter estimates for the two countries, reported in Table 7.

Figure 6: Policy Experiment (Spain)



(a) Profit tax rate

(b) Output and Productivity

7 Conclusion

Motivated by cross-country differences in equity market depth and the relative size and productivity of public and private firms within countries, we examine the importance of public equity markets to aggregate productivity. We start by documenting that public firms are larger, more productive, and better capitalized than private firms in major European countries (France, Germany, Italy, Spain). We also document large increases in firm-level assets and employment around IPOs. These facts point to large differences in access to capital across public and private firms.

To investigate the aggregate implications of these facts, we develop a novel model of firm dynamics that incorporates both public and private firms as well as IPO decisions. Firms enter the economy as private firms that invest in improving productivity and may later become public through an initial public offering. Public firms face lower capital costs than private firms and capital costs that scale less with productivity. We use the model to examine the cross-country importance of public equity markets. We find that the value of public equity markets ranges from around 0.5% of GDP in Italy to around 6% of GDP in France. Equity markets explain between half and three quarters of the productivity differences between France and either Italy or Spain. Within-country differences between public and private firms are also quantitatively important. Lowering capital costs of private firms to match public firms would increase productivity in France, Germany, and Spain by between 19 and 35%. Finally, we find that policies that subsidize IPOs can improve aggregate productivity, despite distorting firm profits and consequently hinder investment, as the overall base of firms that benefit from this policy is small implying a low fiscal burden.

We view our analysis as a first step in linking models with rich firm dynamics to public equity markets and see a number of paths for future research. First, the model could be extended to include more traditional entrepreneur dynamics to fully describe the transition of small to medium to large firms. This could be useful for examining the macro impact of wide-ranging real-world policies supporting small and medium enterprises. Second, the model could be extended to examine the macro impact of differences between public and private firms found in the corporate finance literature, such as differences in investment behaviour of public and private firms (e.g., Bernstein, 2015; Asker et al., 2015).

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A Data Appendix

A.1 Orbis Data Construction

In this subsection we provide additional details about how we clean and construct the Orbis data for the empirical portions of this project.

Multiple observations. The Orbis dataset contains some firm-year observations that are reported in multiple ways within a year. For these firm-year observations, we select the relevant observation according to:

- **Consolidated financial records**. Firms may report financial records for either unconsolidated, consolidated, or both. In the case where both are reported, we use the unconsolidated records.
- **Filing type**. Firms may report financial records as "annual reports" or "local registry filings". In the case where both are reported, we use the annual reports.
- **Month**. Firms may report financial records using different fiscal years. For firms that report multiple fiscal years, we use whichever fiscal year the firm reports most commonly over our sample.

Dropped observations We drop observations according to:

- **Active firms.** We include only firms that are reported as active in the current year, dropping firms with unknown status, in bankruptcy, dissolved, in liquidation, or inactive.
- **Sectors.** We drop firms that operate in the utilities (4900-4999) and financial (6900-6999) sectors. Additional, we drop firms that are reported as being a Bank, Financial Company, Foundation/Research Institute, Insurance Company, Mutual and Pension Fund/Nominee/Trust, Private Equity Firm, Public Authority/State/Government, or Venture Capital.
- **Non-Employers** We drop firms that we never observe employing any workers.

Variable construction. We construct firm-level output as sales less the costs of materials, or zero if the cost of materials exceeds sales in a year. Sales is used when reported and operating revenue when sales is not reported. For assets, we use the firm's book value of fixed assets.

Employment is measured as the count of employees. When number of employees is unreported, we infer it using the wage bill and a constructed sector-year average wage rate. Specifically, inferred employment is $\hat{\ell}_{i,t} = W_{i,t}/\bar{w}_{h,t}$ where $W_{i,t}$ is the wage bill for firm-year (i,t) and $\bar{w}_{j,t}$ is the sector-year (j,t) average of firm wage bill divided by number of employees, for firms in sector j.

Additional cleaning. We winsorize all variables at the top and bottom 2% in each year and for public and private firm types separately.

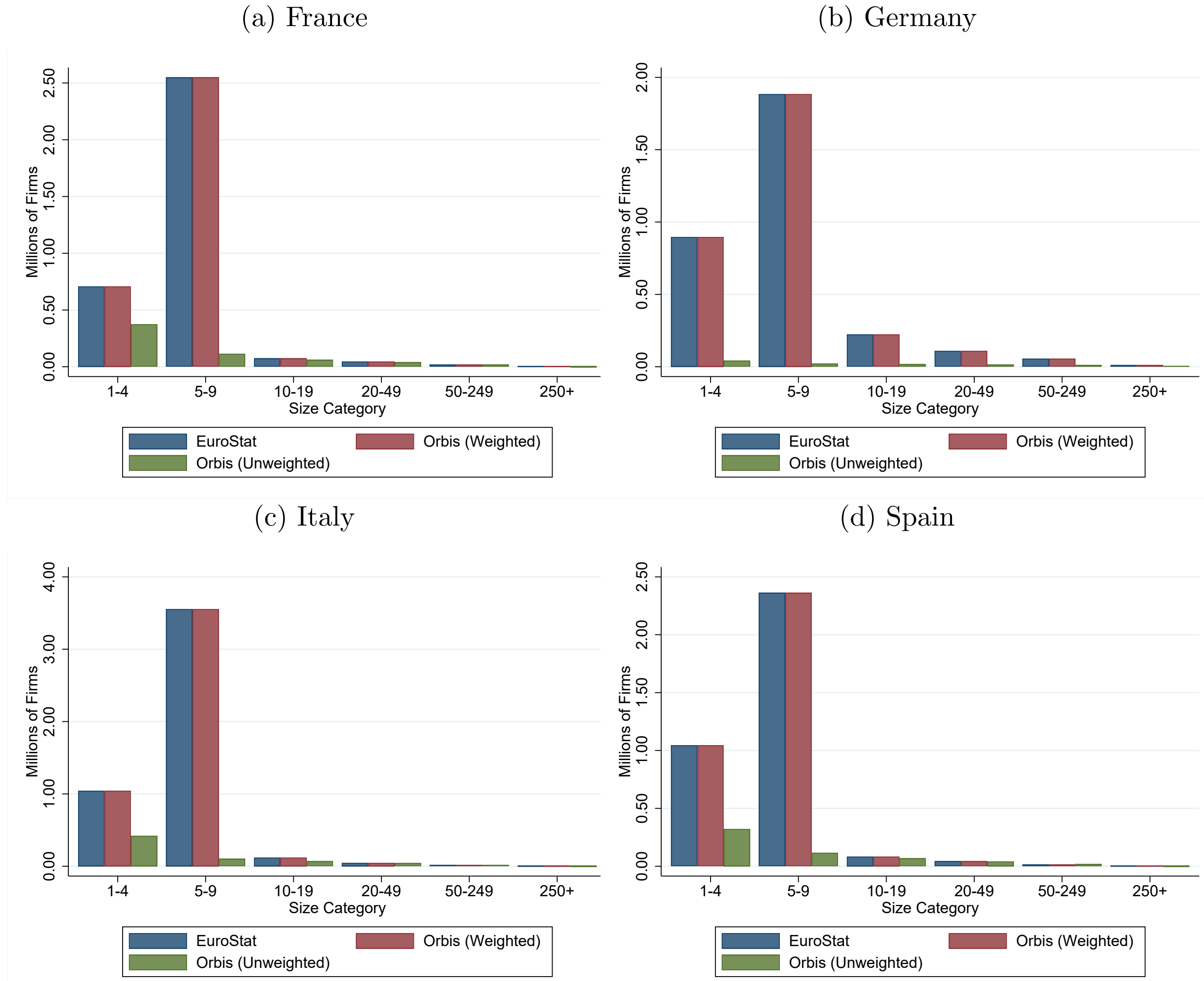
Weights. While Orbis has broad coverage of both public and private firms, it is not a census survey. In order to ensure our firm-size distribution reflects the aggregate numbers, we weight private firms based on their employment size in order to match the distribution of firm sizes in Eurostat data. Let $h_{n,c,t}$ be the share of firms in size bin n in country c in year t. The size bins that we use are reported in Table 12.

Table 12: Eurostat Size Bins

A. BD (h_n^{BD})		B. SBS (h_n^{SBS})		C. Constructed (h_n)		
Bin	Emp.	Bin	Emp.	Bin	Emp.	Definition
0	0					
1	[1,4]	1	[0,9]	1	[1,4]	$h_{[1,4]} = \frac{h^{SBS}_{[0,9]} - h^{BD}_0}{1 - h^{BD}_0} \times \frac{h^{BD}_{[1,4]}}{h^{BD}_{1,4} + h^{BD}_{[5,9]}}$
2	[5,9]			2	[5,9]	$h_{[5,9]} = \frac{h^{SBS}_{[0,9]} - h^{BD}_0}{1 - h^{BD}_0} \times \frac{h^{BD}_{[5,9]}}{h^{BD}_{1,4} + h^{BD}_{[5,9]}}$
3	[10]+	3	[10,19]	3	[10,19]	$h_{[10,19]} = \frac{h^{SBS}_{[10,19]}}{1 - h^{BD}_0}$
		4	[20,49]	4	[20,49]	$h_{[20,49]} = \frac{h^{SBS}_{[20,49]}}{1 - h^{BD}_0}$
		5	[50,249]	5	[50,249]	$h_{[50,249]} = \frac{h^{SBS}_{[20,49]}}{1 - h^{BD}_0}$
		6	$[250,\infty)$	6	$[250,\infty)$	$h_{[250,\infty)} = \frac{h^{SBS}_{[250,\infty)}}{1 - h^{BD}_0}$

We use both the Business Demography (BD) and Structural Business Statistics (SBS) datasets from Eurostat to construct the weights. The BD dataset reports more granular data for smaller business sizes and separates non-employer businesses. The SBS dataset is more granular at higher employment levels. Table 12 reports the size bins constructed in our final dataset. The constructed firm shares exclude non-employer firms. Figure 7 compares the number of firms in each size bin that are reported in the Eurostat data with the number of unweighted firms in Orbis and the number of firms after we apply this weighting scheme.

Figure 7: Data Coverage: Number of Firms



Number of firms reported in EuroStat, in Orbis and in Orbis after applying our weights. All values are averages of years 2000-2019.

We extend the firm shares $h_{n,t}$ to earlier years, when Eurostat data is unavailable, by assuming that the firm share in these periods is the same as in the earliest observation. For example, assume that the earliest period that $h_{n,t}$ can be constructed using the Eurostat data is for 2009, then the firm share for 2005 would be set to $h_{n,2005} = h_{n,2009}$. Similarly, we extend the firm share in the most recent observation $h_{n,t}$ to later periods. We also use the data to interpolate missing observations that occur within the sample. The firm weights are constructed as the relative frequency of firms in size bin n in Eurostat divided by the relative frequency of firms in size bin n in the cleaned Orbis dataset.

A.2 Additional Summary Statistics

Table 13: Output

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	141	491	1,248	737	245,082	492,622
Germany	276	2,106	17,354	8,366	281,550	26,279
Italy	108	503	1,619	838	165,746	442,960
Spain	53	245	706	384	446,699	472,986

Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data. Values are in thousands of USD.

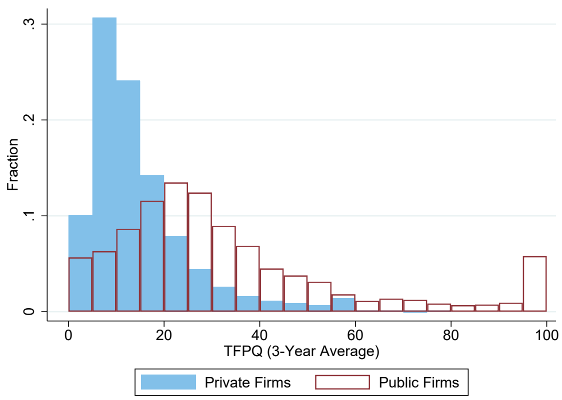
Table 14: Assets

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	7	92	697	352	1,076,983	631,943
Germany	5	100	5,147	4,588	271,521	62,357
Italy	9	115	1,140	558	396,459	474,477
Spain	6	97	986	516	1,117,108	568,992

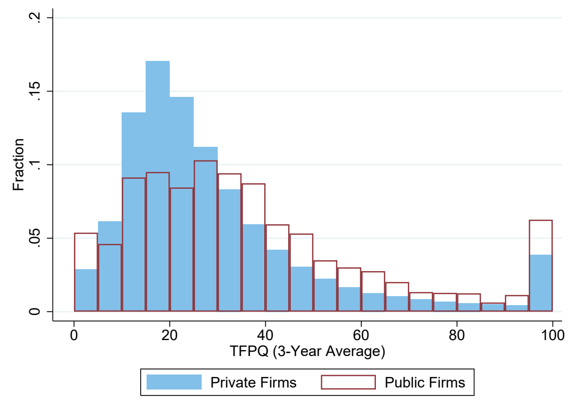
Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data. Values are in thousands of USD.

Figure 8: TFPQ (3-Year Average)

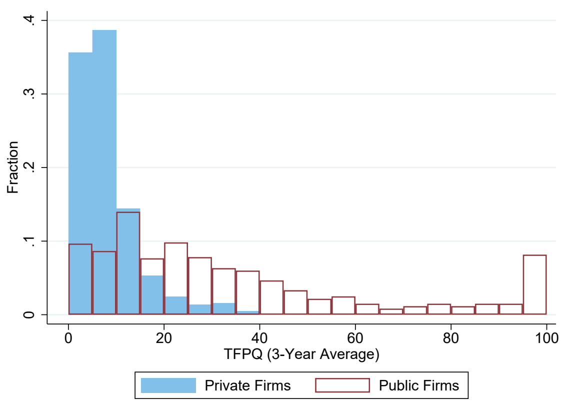
(a) France



(b) Germany



(c) Spain



(d) Italy

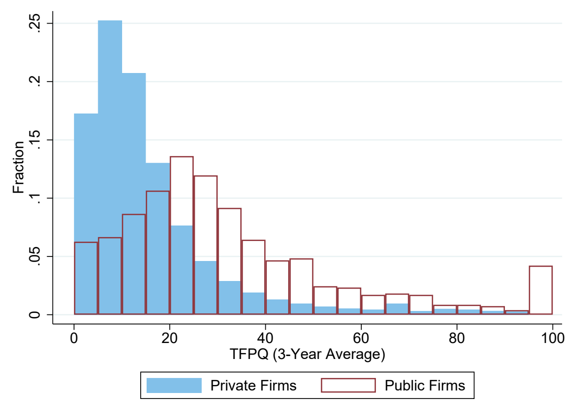
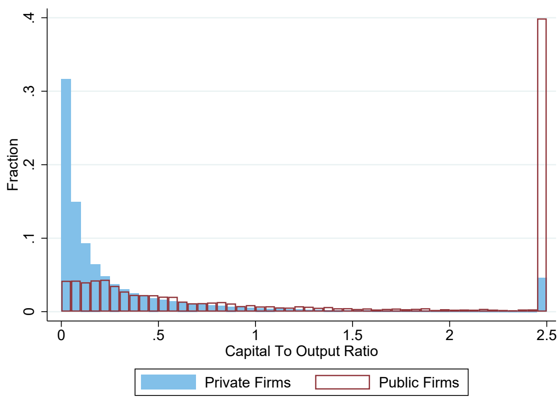
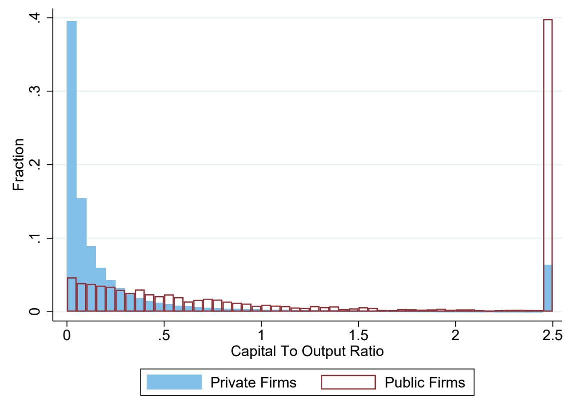


Figure 9: Capital To Output Ratios

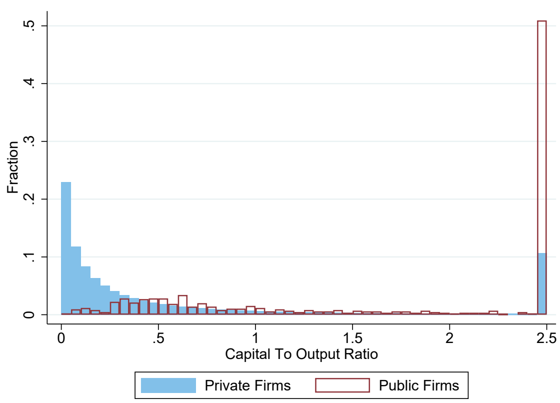
(a) France



(b) Germany



(c) Spain



(d) Italy

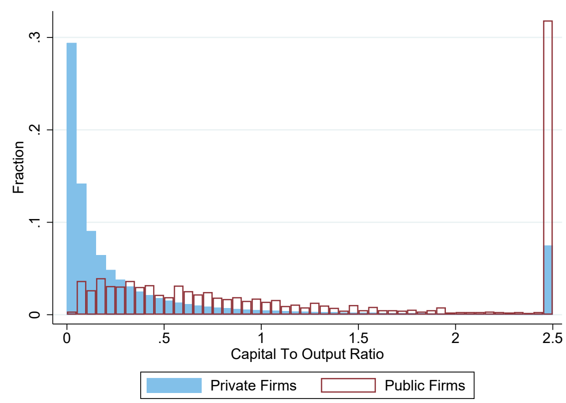


Table 15: Sales per Employee

	Percentiles			Private Firm	Public Firm	Number of
Country	P10	P50	P90	Average	Average	Firms
France	55	130	419	205	776	256,974
Germany	54	177	1,188	500	705	45,767
Italy	51	175	704	311	570	368,610
Spain	33	90	347	160	761	497,600

Orbis data. All values, including the number of firms, are averages across sample years. Data on private firms is weighted to match the distribution of firm sizes in EuroStat data. Values are in thousands of USD.

Table 16: French Summary Statistics

	Employment	Sales	Assets	$\frac{\text{Sales}}{\text{Employees}}$
		Thousands of USD		
All Firms				
Mean	8	1,891	2,108	207
p10	1	174	7	55
p50	6	743	92	130
p90	9	2,533	697	419
Avg Obs per Year	631,007	631,893	631,943	256,974
Private Firms				
Mean	6	1,355	352	205
p10	1	173	7	55
p50	6	742	92	130
p90	9	2,510	687	417
Avg Obs per Year	630,619	631,506	631,555	256,674
Public Firms				
Mean	1,376	334,328	1,076,983	776
p10	131	19,538	8,526	97
p50	721	157,516	103,595	285
p90	4,067	1,092,235	3,214,107	1,185
Avg Obs per Year	387	387	388	300

All values are averages across sample years. All variables are winsorized below the 2^{th} and above the 98^{th} percentiles. Data on private firms is weighted to match the distribution of firm sizes in Eurostat data.

A.3 Measuring Wedges in the Data

One way to evaluate the differences in capital access across private and public firms is with the following regression specification:

$$\log(\text{Capital Wedge}^k_{j,t}) = \beta_0 + \beta_1 \mathbb{I}^{\text{Public}}_{j,t} + \beta_2 \mathbb{I}^{\text{Public}}_{j,t} \log(TFP_{j,t}) + \beta_3 \log(TFP_{j,t}) + \alpha_s + \alpha_t + \epsilon_{j,t} \tag{21}$$

where Capital $\text{Wedge}^k_{j,t} = \frac{\text{Sales}_{j,t} - \text{Material Costs}_{j,t}}{Assets_{j,t}}$, $\mathbb{I}^{\text{Public}}_{j,t}$ is an indicator variable equal to one if firm j is public in year t, α_s is a set of sector fixed effects and α_t is a set of year fixed effects.

We plot the implied relationship between log TFP and log Capital $\text{Wedge}^k_{j,t}$ in figure 10. If the allocation of capital was perfectly efficient, we should expect this relationship to be flat. Firms with different levels of TFP should have the same expected marginal product of capital if they all faced the same cost of capital. Our results find β_3 to be significantly positive, which suggests some degree of capital misallocation. High TFP firms have, on average, higher marginal products of capital than low TFP firms. This is consistent with the idea that financial frictions, like those in our model, constrain the ability of high TFP firms to expand their scale and take advantage of their high marginal product of capital.

Across the distribution of productivity, public firms have lower measured marginal product of capital. This suggests that these public firms have much better access to capital, as is consistent with the results from the IPO regressions in section 2.3.

In addition, in France, Germany, and Spain β_2 is significantly negative. This implies that highly productive public firms are able to attract relatively more capital due to their high productivity than highly productive private firms. This is not true in Italy, where our estimate of β_2 is positive, which suggests that highly productive public firms do not gain access to relatively more capital than highly productive private firms. That being said, the implied capital wedges for Italian public firms is still consistently below the implied capital wedges for Italian private firms.

As a validation exercise, we can also run the analogous regression to identify a labour wedge:

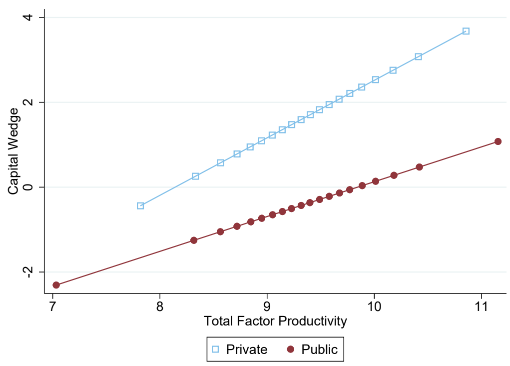
$$\log(\text{Labour Wedge}^k_{j,t}) = \beta_0 + \beta_1 \mathbb{I}^{\text{Public}}_{j,t} + \beta_2 \mathbb{I}^{\text{Public}}_{j,t} \log(TFP_{j,t}) + \beta_3 \log(TFP_{j,t}) + \alpha_s + \alpha_t + \epsilon_{j,t} \tag{22}$$

where Labour $\text{Wedge}^k_{j,t} = \frac{\text{Sales}_{j,t} - \text{Material Costs}_{j,t}}{\text{Employment}_{j,t}}$, $\mathbb{I}^{\text{Public}}_{j,t}$ is an indicator variable equal to one if firm j is public in year t, α_s is a set of sector fixed effects and α_t is a set of year fixed effects.

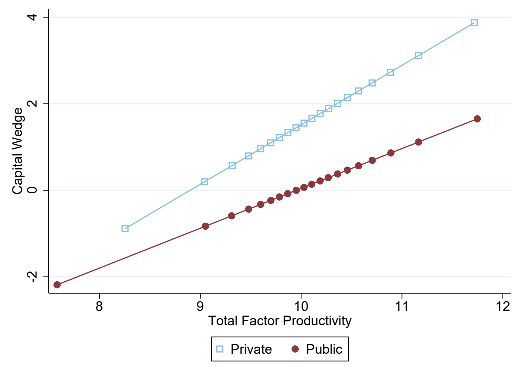
We plot the implied relationship between log TFP and log Labour $\text{Wedge}^k_{j,t}$ in figure 11. Again, if the allocation of labour was perfectly efficient, we should expect this relationship to

Figure 10: Capital Wedges and TFP

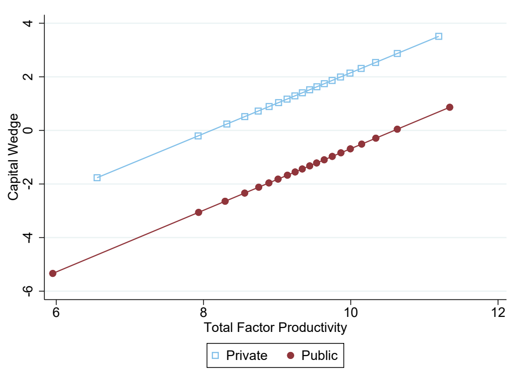
(a) France



(b) Germany



(c) Italy



(d) Spain

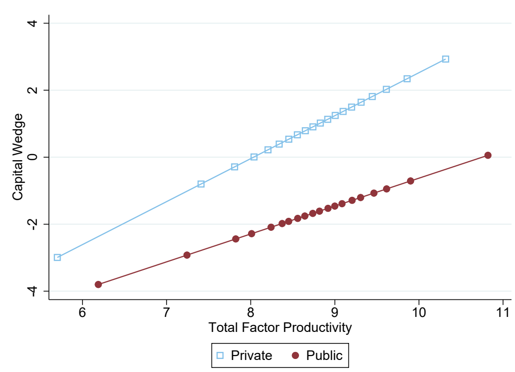
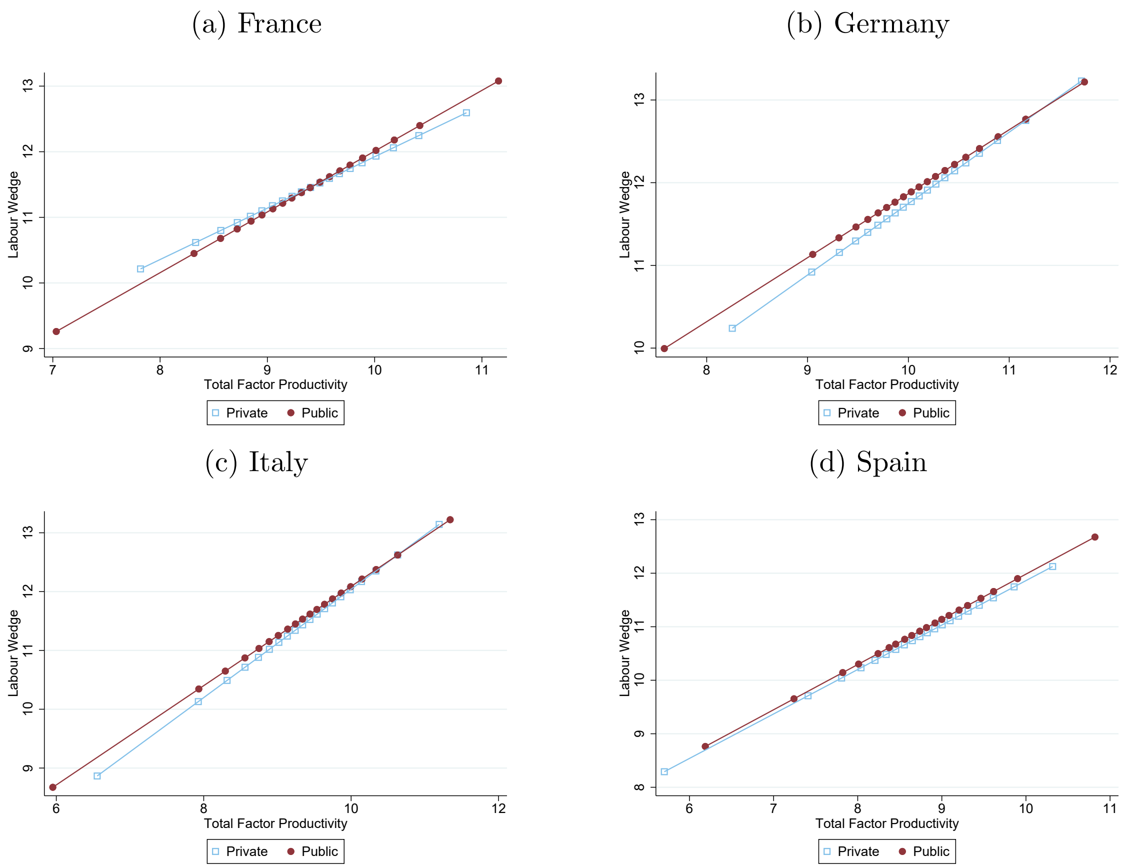


Figure 11: Labour Wedges and TFP



be flat. As with capital, we find that β_3 is significantly positive, which suggests some degree of misallocation of labour. However, we see that the implied differences between public and private firms in terms of the allocation of labour are quantitatively small, indicating that the degree of misallocation of labour between public and private firms is minimal. This contrasts with figure 10, which shows large differences in the capital wedge across public and private firms, suggesting a high degree of misallocation of capital between public and private firms.

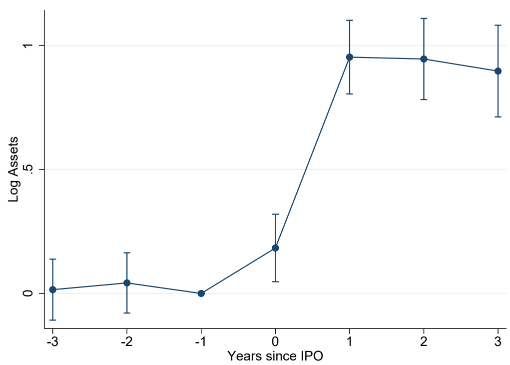
A.4 IPO Results Appendix

A.4.1 Benchmark

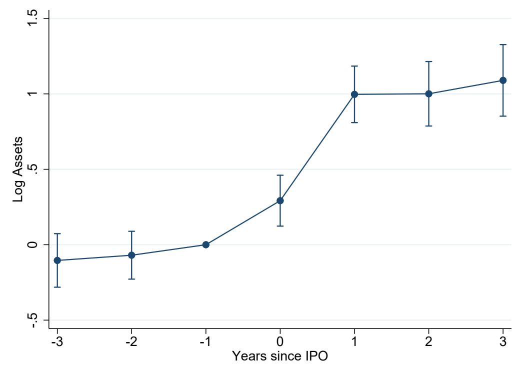
Here we report the results for estimating the benchmark equation 1 across the four countries.

Figure 12: Increase in Log Assets

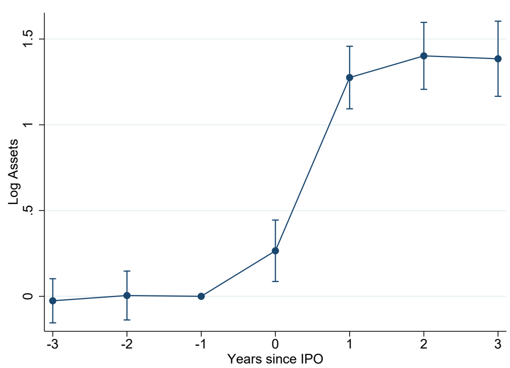
(a) France



(b) Germany



(c) Italy



(d) Spain

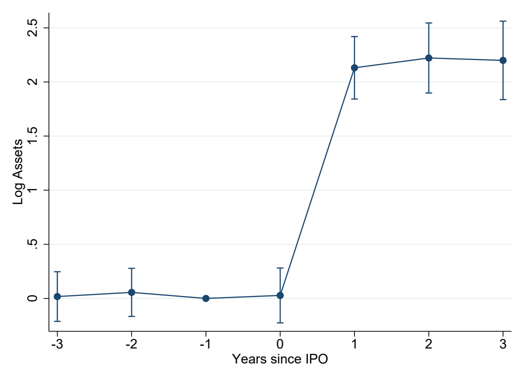
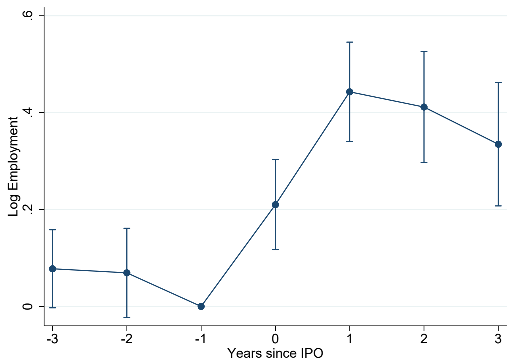
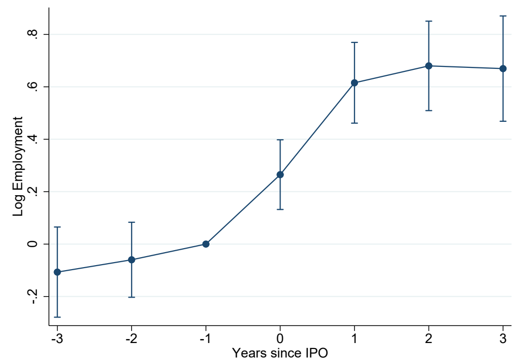


Figure 13: Increase in Log Employment

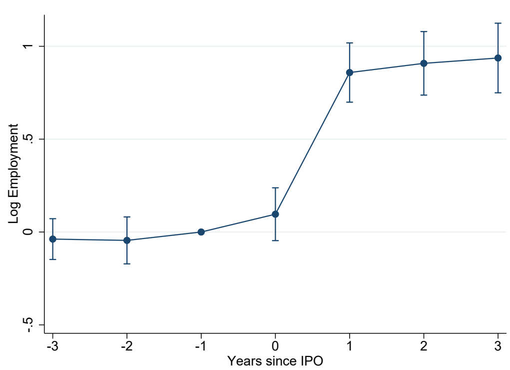
(a) France



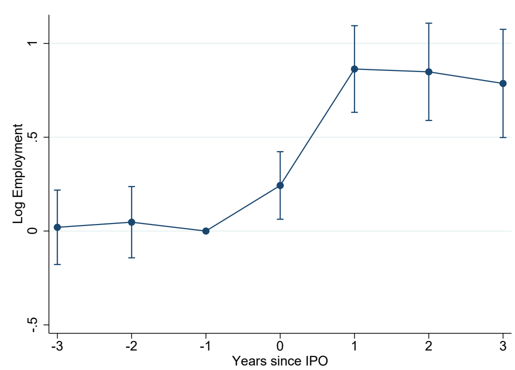
(b) Germany



(c) Italy



(d) Spain



A.4.2 Relative Employment

An alternative specification to equation 1 is the following:

$$\frac{y_{i,t}}{y_{i,k=-1}} = \sum_{\substack{k=-3 \\ k\neq-1}}^{3} \beta_k D_{i,t}^{k} + \alpha_i + \alpha_t + \epsilon_{i,t} \tag{23}$$

where $\frac{y_{i,t}}{y_{i,k=-1}}$ is the outcome of interest y for firm i in year t relative to its value in the year before IPO, which we denote as $k = -1$. α_i is a vector of sector fixed effects, α_t is a vector of year fixed effects, and $\epsilon_{i,t}$ is an error term. The coefficients of interest are a series of coefficients $\{\beta_k\}$, on dummy variables $D_{i,t}^{k}$. These variables equal one if firm i conducted an IPO in year $t - k$, and are zero otherwise.

Figure 14: Relative Assets over an IPO

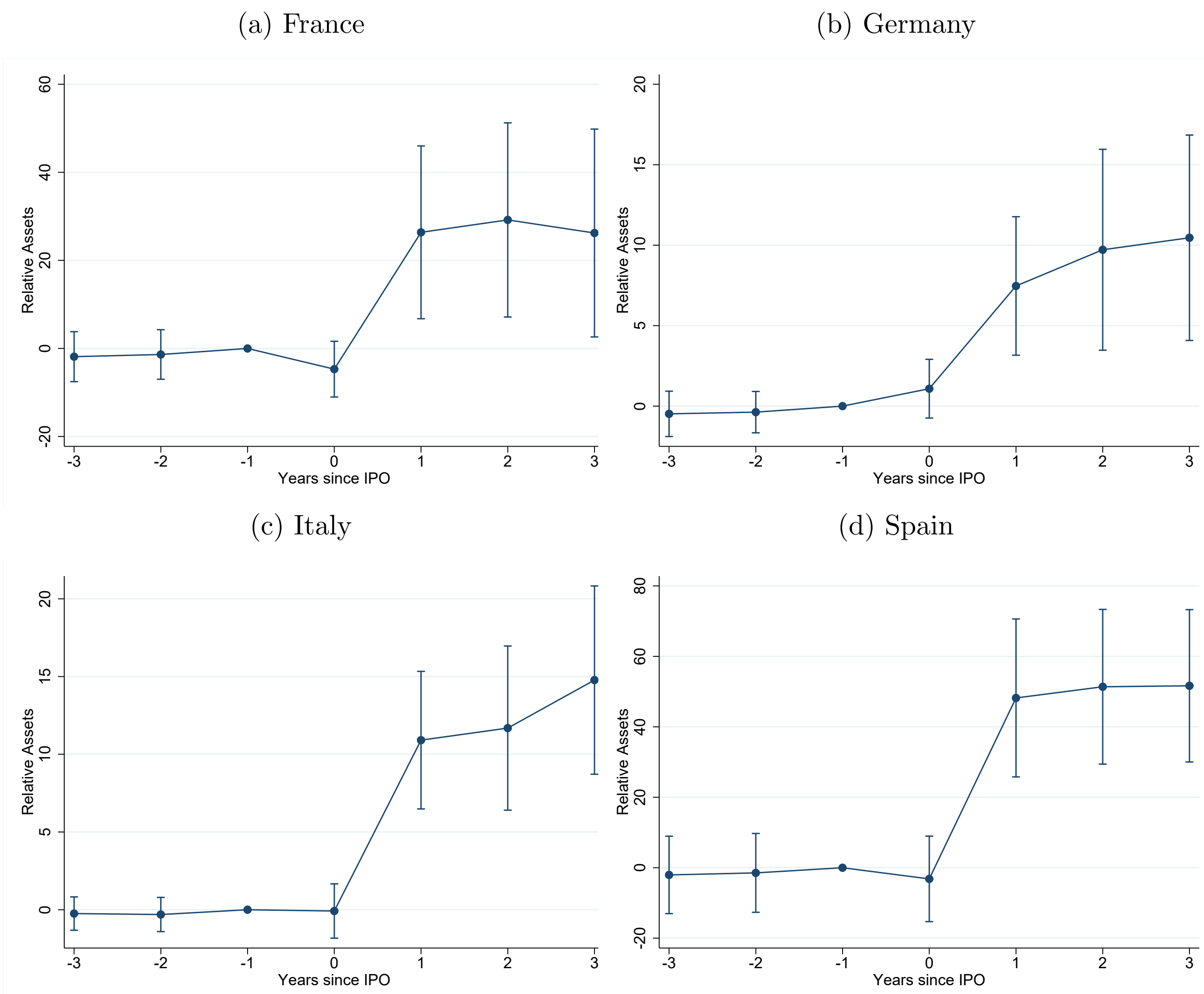
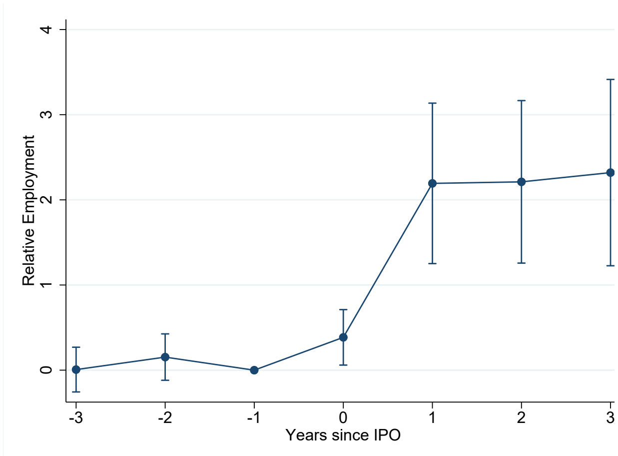
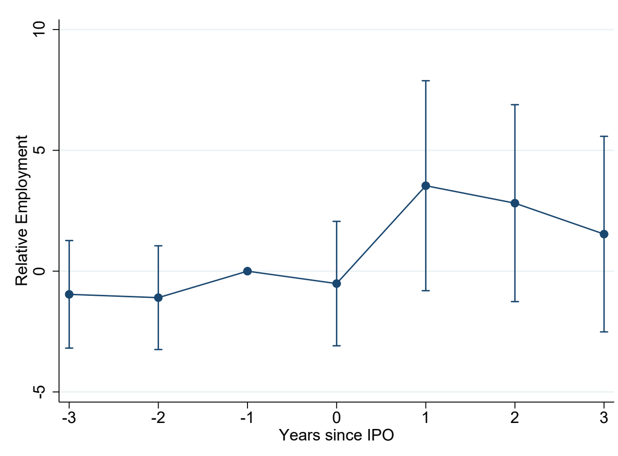


Figure 15: Relative Employment over an IPO

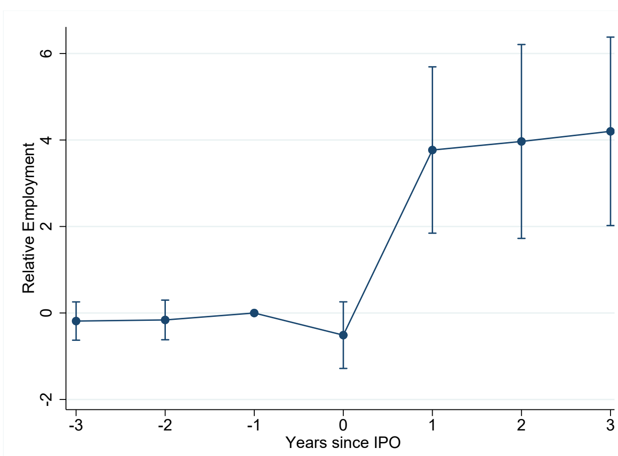
(a) France



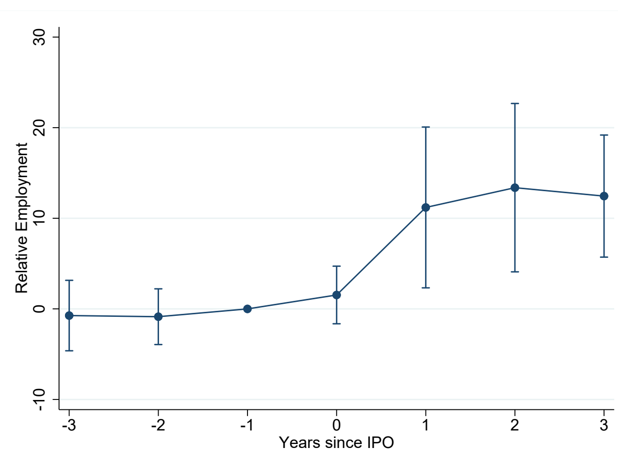
(b) Germany



(c) Italy



(d) Spain

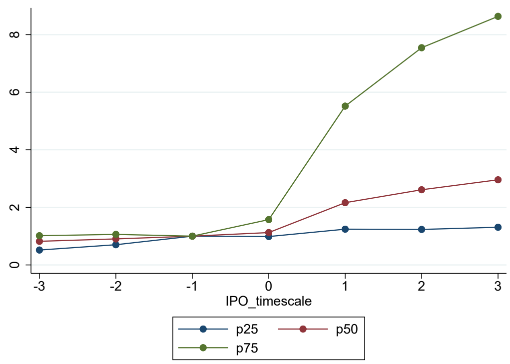


A.4.3 Percentiles

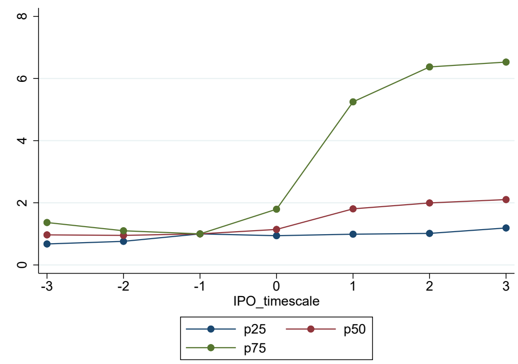
Finally, in order to study the heterogeneity in the effect of IPOs, we plot percentiles of the distribution of assets and employment relative to the year before IPO.

Figure 16: Increase in Relative Assets - Percentiles

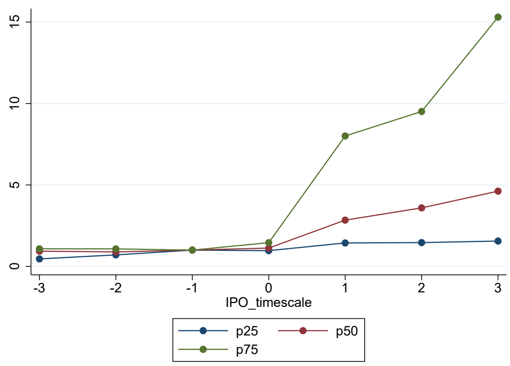
(a) France



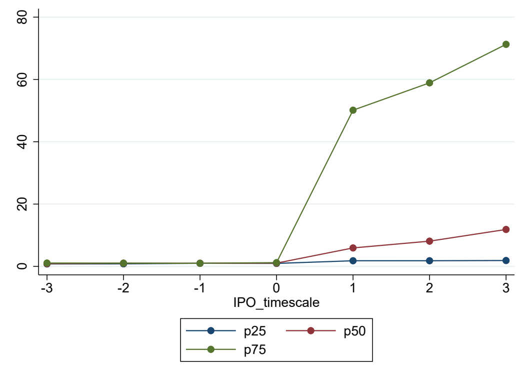
(b) Germany



(c) Italy



(d) Spain

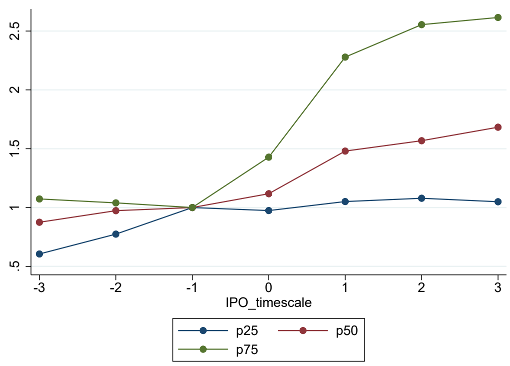


These percentiles show that some firms experience tremendous growth. The 75th percentile firm in France increases their assets by more than 8 times their pre-IPO asset level. In all four countries, the median firm that goes through an IPO at least triples their asset level by the 3rd year after going through an IPO.

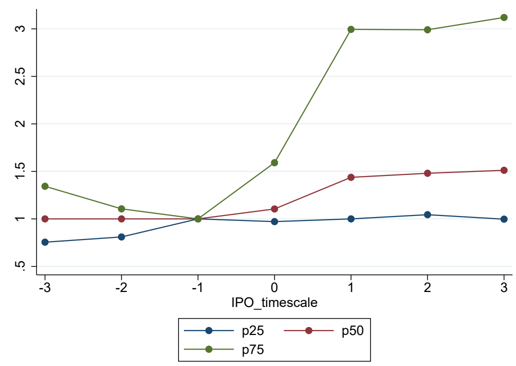
Employment growth is less dramatic, but still the median firm going through an IPO in every country experiences employment growth of at least 50% by the third year following an IPO.

Figure 17: Increase in Relative Employment - Percentiles

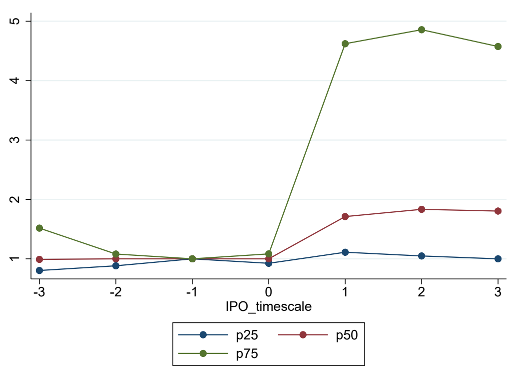
(a) France



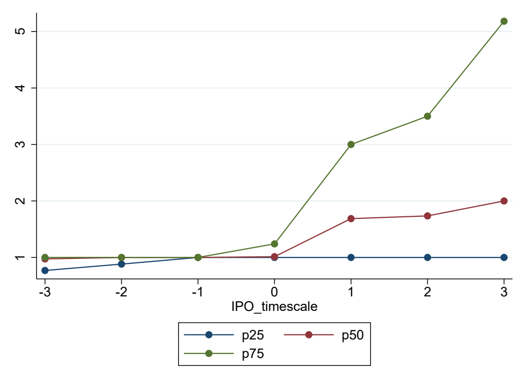
(b) Germany



(c) Italy



(d) Spain

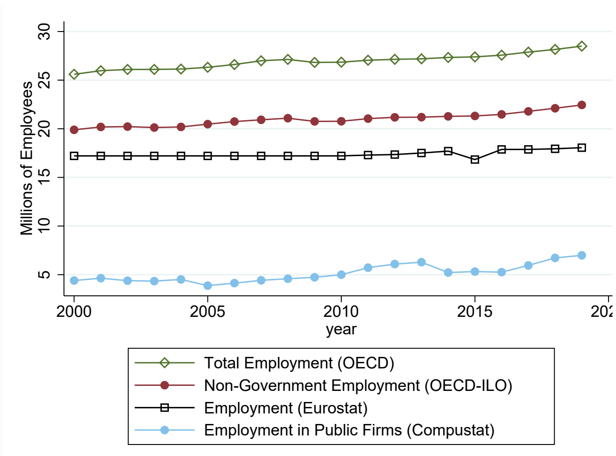


A.5 Measuring Public Firm Employment Shares

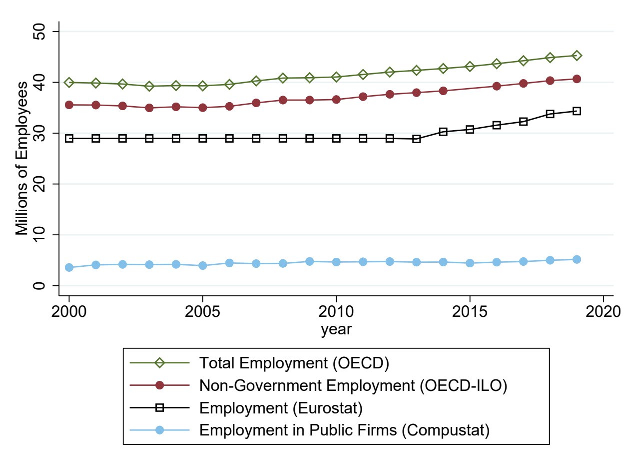
An additional measure of the depth of public equity markets is the proportion of the labour force in a country that works for publicly listed firms, which we call the public firm employment share. In the absence of data on the publci firm employment share, we estimate the public firm employment share for the four countries that are the focus of our paper. To do so, we follow Asker et al. (2015), who compute the public firm employment share as the sum of employment in public firms in Compustat and divide this by non-government employment in the National Income and Product Accounts.

Figure 18: Employment by Year

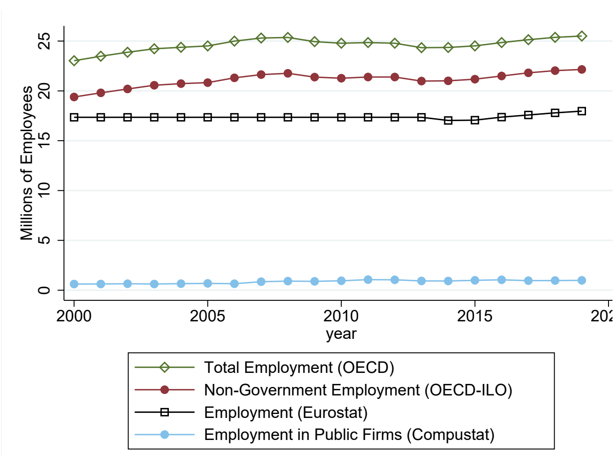
(a) France



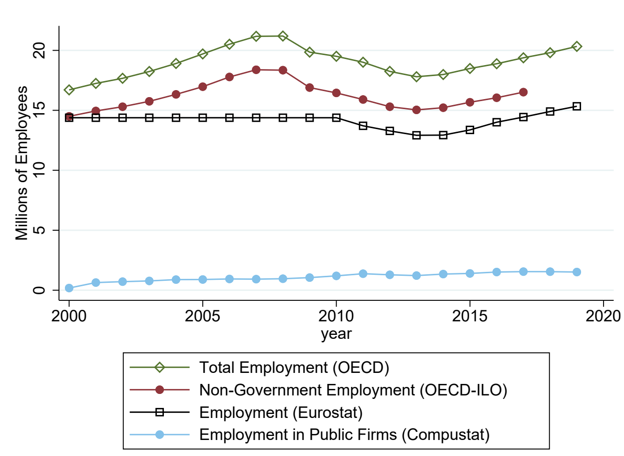
(b) Germany



(c) Italy



(d) Spain

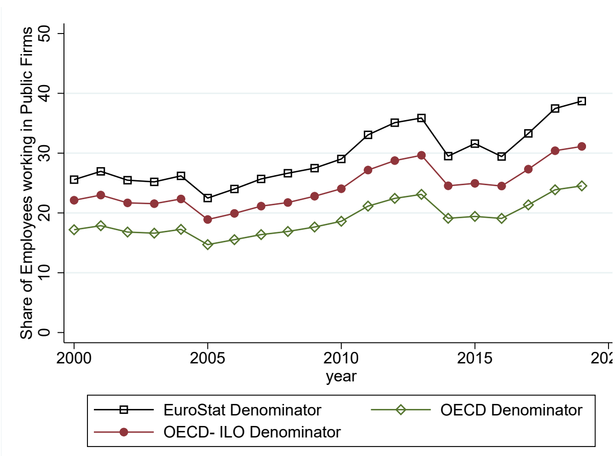


We use Compustat global to construct a measure of employment in public firms within a country in any given year. We then need a measure of the total labour force. Ideally, we would like a measure of the labour force excluding government employees, as we want the public firm employment share to pick up variation in the proportion of the labour force that works for public vs. private firms rather than variation in the share of the labour force working for government organizations. We therefore use total employment in a country from the OECD and subtract a measure of total employment for government organizations that we obtain from the International Labour Organization (ILO), a UN agency. As an alternative measure, we also use total employment as calculated in the EuroStat firm-level data.

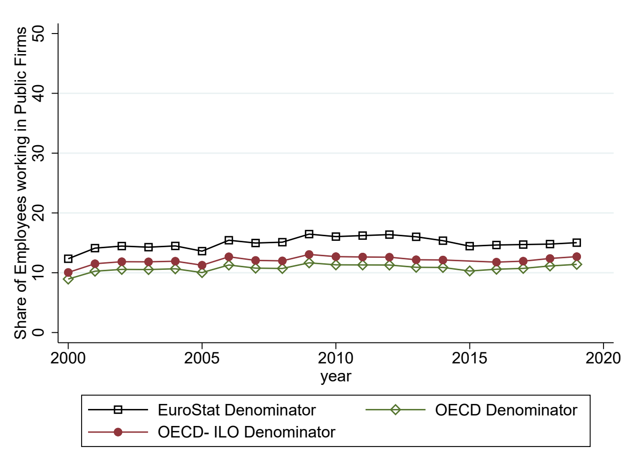
Figure 18 shows the number of employees over time across these different data sources. Figure 19 plots our estimates for the public firm employment share over our sample period 2000-2019 for the three countries. We plot three different measures, constructed with three different denominators. Regardless of which measure we consider, France has the highest proportion of the labour force working for public firms while Italy has the lowest. Our preferred measure, which we use in the calibration in section 4 is the one with a denominator constructed as the total employment from OECD minus government employment in ILO.

Figure 19: Public Firm Employment Share by Year

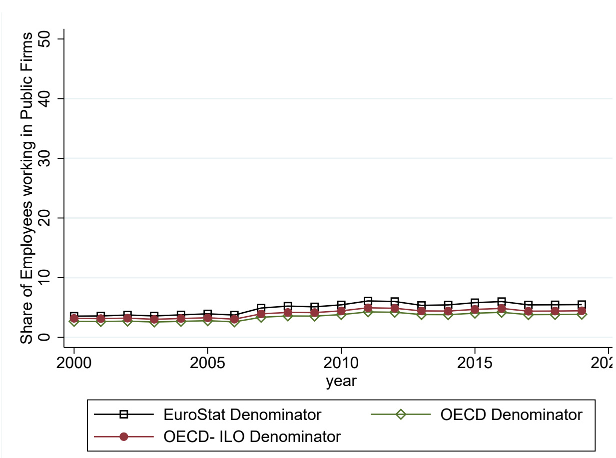
(a) France



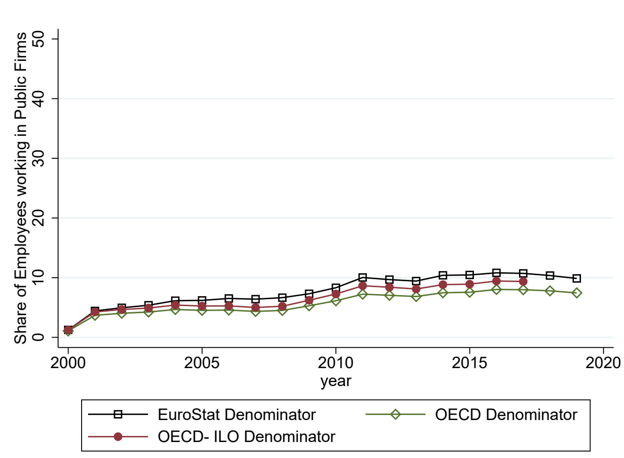
(b) Germany



(c) Italy



(d) Spain



A.6 Additional Cross-Country Evidence

In this subsection, we provide some additional cross-country evidence on the importance of public equity market depth for total factor productivity. We run the following cross-country regression:

$$\text{TFP}_{i,t} = \beta_0 + \beta_1 \text{Public Equity Market Depth}_{i,t} + \beta X_{i,t} + \epsilon_{i,t} \tag{24}$$

where $\text{TFP}_{i,t}$ is total factor productivity for country i in year t, which we calculate in the Penn World Tables. Public Equity Market $\text{Depth}_{i,t}$ is the market capitalization of public firms divided by GDP in country i in year t, which we obtain from the World Bank. $X_{i,t}$ is a vector of other financial variables we take from the IMF.

Table 17: Cross-Country Public Equity Market Depth Regression

	(1) TFP	(2) TFP
EquityMktDepth	0.999***	0.352***
	(0.0766)	(0.0722)
DomCredit		-0.724**
		(0.280)
PrivateDebt		6.167***
		(0.528)
NonFinancialDebt		-5.041***
		(0.698)
Constant	834.5***	729.5***
	(8.945)	(20.34)
N	2229	949

Table 17 shows that across-countries and time periods public equity market depth is correlated with total factor productivity. This correlation weakens, but does not disappear when other measures of financial development are included, like the total amount of domestic credit, the total amount of private debt and the total level of non financial debt.

B Model Appendix

B.1 One-Period Model

We consider a simplified version of the baseline quantitative model to highlight the model's theoretical predictions.

Economic environment. The economy is populated by a unit mass of firms where an initial share μ are public and the rest are private. The period's timing is as follows: (i) firms invest in improving their production technology; (ii) firms realize their productivity z; (iii) firms make IPO decisions; and (iv) firms make production decisions and earn profits. Firm production depends only on productivity, which equals $y = z$ for private firms and $y = z + \eta$ for public firms, where η captures differences in efficiency of public and private firms. Private firms earn profits $(\pi - \tau)z$ and public firms earn profits πz, where $\tau > 0$ captures the impact of distortions and efficiency differences on profits. Firms pay a fixed IPO cost f to go public implying that firms will go public if $\tau\pi z > f$. Firm productivity and profits increase by proportion $\lambda > 1$ with probability x chosen by the firm through investing $x^2\psi/2$.

Equilibrium. We assume that parameters are such that a private firm that does not improve technology never finds it profitable to go public. The investment problem of a private firm is then to maximize expected net profits according to

$$\max_x \left[x \max\left\{\pi\lambda - f, (\pi - \tau)\lambda\right\} + (1 - x)(\pi - \tau)\right] - \frac{\psi}{2}x^2,$$

where profits in the next period depend the success of the firms investment and, if successful, whether a firm chooses to go through with an IPO. The problem of a public firm is

$$\max_x \left[x\pi\lambda + (1 - x)\pi\right] - \frac{\psi}{2}x^2.$$

Investment is then given by

$$x^{Pri} = \frac{(\pi - \tau)(\lambda - 1) - \min\{f - \tau\lambda, 0\}}{\psi}$$
$$x^{Pub} = \frac{\pi(\lambda - 1)}{\psi}$$

and the share of IPOs is

$$\Delta\mu = (1 - \mu)x^{Pri}1_{\tau\lambda > f}$$

where $1_{\tau\lambda>f}$ takes value one if $\tau\lambda > f$ and zero otherwise.

Aggregate output depends on the distribution firms across states in the economy. Aggregate output is given by

$$
\begin{aligned}
Y &= \int_j y_j dj \\
&= \mu \left[x^{Pub}\lambda(z+\eta) + (1-x^{Pub})(z+\eta)\right] + (1-\mu)\left[x^{Pri}\lambda(z+1_{\tau\lambda>f}\eta) + (1-x^{Pri})z\right].
\end{aligned}
$$

Comparative statics. Our main outcome of interest is aggregate output Y, which depends on the number of IPOs $\Delta\mu$ and investment by public and private firms x^{κ}. We focus on the impact of four parameters: (i) fixed IPO costs f, (ii) the technological efficiency η, (iii) the profitability wedge τ, and (iv) the public firm share μ. Table 18 summarizes the comparative statics in the one-period model.

Table 18: Comparative Statics in the One-Period Model

	dx^{Pri}	$d\Delta\mu$	dY
df	$-\frac{IPO(f,\tau)}{\psi} \leq 0$	$\frac{dx^{Pri}}{df} \leq 0$	$(1-\mu)[\lambda(z+IPO(f,\tau)\eta)-1]\frac{dx^{Pri}}{df} \leq 0$
$d\eta$	0	0	$\mu[x^{Pub}\lambda + (1-x^{Pub})] + (1-\mu)x^{Pri}\lambda(IPO(f,\tau)) \geq 0$
$d\tau$	$-\frac{\lambda-(1-IPO(f,\tau))}{\psi} \lesseqgtr 0$	$1_{\tau\lambda>f}\frac{dx^{Pri}}{d\tau} \lesseqgtr 0$	$(1-\mu)(\lambda(z+IPO(f,\tau)\eta)\frac{dx^{Pri}}{d\tau} \lesseqgtr 0$
$d\mu$	0	$-x^{Pri}1_{\tau\lambda>f} \leq 0$	$\left[x^{Pub}\lambda(z+\eta)+(1-x^{Pub})(z+\eta)\right] - \left[x^{Pri}\lambda(z+1_{\tau\lambda>f}\eta)+(1-x^{Pri})z\right] \lesseqgtr 0$

Notes: The values report the derivative of the first row with respect to the first column. $IPO(f,\tau) = 1_{\tau\lambda>f} + J(f,\tau)$ *where* $1_{(f,\tau)}$ *takes values 1 if firms engage in IPOs* ($\tau\lambda > f$) *and J is a dirac delta function with mass at the IPO cutoff for each parameter.*

Higher IPO costs f can discourage private firm investment, if IPOs are viable, and lower aggregate output. Higher public efficiency generally increases aggregate output depending on the existing share of public firms μ and the relative investment intensity of public and private firms, where the latter only matters if IPOs are viable. Higher private wedges τ tends to discourage private investment, lower IPOs, and decrease aggregate output. The inequality switches at the cutoff of IPO viability. Increasing the private wedge increases investment when IPOs are viable since increasing the private wedge increases the relative gain in profits from successfully improving the production technology. By similar arguments, aggregate output also increases when IPOs are viable. Finally, increasing the public firm share μ leads to fewer IPOs, because of fewer private firms, and has an ambiguous impact on aggregate output that depends on the relative static through η and dynamic efficiency through the relative values of x^{Pri} and x^{Pub}.

Relationship with quantitative model. The one-period model helps highlight the key economics of the more complex quantitative model developed in Section 3. Parameters are labeled to correspond to the those in the quantitative model, noting that relationships in the full model are more complex. In particular, the technology efficiency η captures a combination of public-specific productivity, previous investment, firm type, and capital wedges in the quantitative model since all of these factors determine the productivity of firms. Similarly, the profitability wedge τ captures a (different) combination of these same factors, which affect firm profitability. Additionally, while we model η and τ as affecting only public and private firms in the one-period model, there are differences by both public and private firms as well as within public and private firm groups in the quantitative model.

C Quantitative Appendix

C.1 Solution Method

Rather than solve the baseline model, we solve an alternative model with a scaling factor (Λ). Specifically, we solve a model with parameters $\psi = \tilde{\psi}\Lambda$ and $f = \tilde{f}\Lambda$ where we set the scaling factor to

$$\Lambda = \left(\frac{\alpha}{r}\right)^{\frac{\alpha\gamma}{1-\gamma}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}}. \tag{25}$$

The importance of the scaling factor Λ is that profits are proportional to this factor. Note that

$$\begin{aligned}\pi^{\kappa}_{a,n}(v) &= \left[\left(\frac{\alpha}{\tau_\kappa r}\right)^{\frac{\alpha\gamma}{1-\gamma}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}}\right] \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma) z^{\kappa}_{a,n}(v) \\ &= \Lambda\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)\tau_\kappa^{-\frac{\alpha\gamma}{1-\gamma}} z^{\kappa}_{a,n}(v)\Lambda.\end{aligned}$$

In equilibrium, the wage rate w and the interest rate r are constant, implying that the solution to the alternative and baseline models are the same with the appropriate rescaling of parameters. The advantage to solving the alternative model is that we find it is computationally more stable than the baseline model, making computational solvers more effective.

C.2 Calibration Moments for Other Country

Tables 19, 20, and 21 report the model and data moments for Germany, Italy, and Spain.

Table 19: Calibration Moments (Germany)

Moment	Data	Model
Entry Rate (%)	7.9	8.2
Exit Rate Elasticity	-0.0011	-0.00477
Avg Emp Growth (%)	3.77	4.20
Std Emp Growth	28.9	18.1
Std Employment	0.91	0.63
Firm Share of Small Firms (%)	79.2	49.2
Emp Share of Small Firms (%)	17.9	25.0
Emp Share of Public Firms (%)	12.0	12.0
Relative Size of Public Firms	29.1	30.3
IPO Capital Growth	1.0	0.8

Table 20: Calibration Moments (Italy)

Moment	Data	Model
Entry Rate (%)	7.1	7.2
Exit Rate Elasticity	-0.0011	-0.00193
Avg Emp Growth (%)	7.06	6.17
Std Emp Growth	38.9	34.8
Std Employment	0.72	0.99
Firm Share of Small Firms (%)	87.8	90.6
Emp Share of Small Firms (%)	35.1	34.7
Emp Share of Public Firms (%)	5.0	5.2
Relative Size of Public Firms	129.2	149.1
IPO Capital Growth	1.3	1.9

Table 21: Calibration Moments (Spain)

Moment	Data	Model
Entry Rate (%)	8.6	9.3
Exit Rate Elasticity	-0.0011	-0.00181
Avg Emp Growth (%)	3.79	3.79
Std Emp Growth	32.9	32.2
Std Employment	0.77	1.01
Firm Share of Small Firms (%)	90.7	91.4
Emp Share of Small Firms (%)	33.1	38.1
Emp Share of Public Firms (%)	10.0	10.0
Relative Size of Public Firms	320.3	301.8
IPO Capital Growth	2.1	1.6

C.3 Sensitivity Analysis

Table 22 reports the sensitivity of model moments to a 10% changes in underlying model parameters. This represents a formalization of the relationship between parameters and model moments.

Table 22: Sensitivity of Model Moments

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
ν	10.0	-1.3	-15.4	-0.2	-12.3	5.4	83.1	-71.3	30.3	-2.2
χ	0.1	10.1	0.1	-0.0	-0.0	-0.0	0.1	-0.1	-0.0	-0.0
λ	-0.0	-7.0	54.5	0.7	23.8	-9.0	-68.2	97.9	-9.1	11.8
ψ	0.0	0.4	-14.1	-0.2	-7.4	3.1	44.4	-42.5	17.0	-1.9
η	0.0	0.2	-1.1	-0.0	-1.3	0.3	10.6	-23.2	0.4	-2.7
a_l	0.0	4.6	-0.0	0.0	-3.5	2.5	9.5	-0.5	-0.7	0.0
ω_{a_l}	0.0	-4.2	-8.9	1.9	-0.9	1.6	2.9	-1.1	8.2	0.0
σ	-0.0	-2.3	0.2	9.2	0.5	-0.3	-1.9	1.4	-0.6	-0.0
f	0.0	0.0	-0.2	-0.0	-0.2	0.0	1.0	-2.6	3.0	0.9
$\overline{TFP}$	-0.0	-5.2	36.2	0.8	18.8	-6.7	-65.2	165.3	-17.3	39.0

Notes: Moments are reported in the same order as Table 5. M1 Entry Rate; M2 Exit Rate Elasticity; M3 Avg Emp Growth; M4 Std Emp Growth; M5 Std Employment; M6 Emp Share of Small Firms; M7 Firm Share of Small Firms; M8 Public Firm Share; M9 Relative Public Size; M10 IPO Capital Growth.

C.4 Other Quantitative Experiment: Importance of Public Productivity

In the main text, we show that lowering capital costs of private firms to match those faced by public firms can substantially increase aggregate productivity. Public and private firms also differ in terms of the residual corporate structure specific component of productivity η, which implies that public firms are, all else equal, less productive. We now consider the aggregate impact of setting $\eta = 1$ in the four calibrated economies. Table 23 reports the results.

The increase in productivity is of a similar magnitude as the lowering capital wedge experiment in Table 11 for France, Germany, and Spain. Italy has the largest potential increase in aggregate productivity from decreasing the loss in productivity faced by public firms, corresponding to an increase in productivity of between 40% to 80%.

Despite public firms accounting for a relatively small share of firms, the change in the public-specific component of productivity has a large aggregate impact. This is driven by two factors. First, public firms account for around one quarter of employment and output

Table 23: Gains from Improving Public Productivity

Parameter	FR	DE	IT	ES
Output (%)	19.7	33.7	84.5	29.2
TFP (%)	21.7	35.6	41.3	30.4
Mass of Firms (%)	1.9	7.0	62.4	1.3
Investment / GDP (pp)	2.2	6.3	5.9	2.7
Firm Share of Large Firms (pp)	0.6	2.6	4.5	0.7
Emp Share of Large Firms (pp)	32.3	55.7	75.7	49.4

Notes: The table reports the percent change (%) and the percentage point change (pp) in outcomes between the counterfactual and benchmark economies. Counterfactual economies set the residual public-specific component of productivity to $\eta = 1$.

produced in the economy, implying much larger benefits from increasing the productivity of public firms than other firms. Second, increasing the public component of productivity causes public firms to become more profitable leading to firms conducting IPOs sooner and investing more both prior to IPOs and after becoming public causing a large increase in investment and the mass of operating firms in the economy.